HYCON2 - BALCON
2-3 JULY 2013, BELGRADE

Non-Asymptotic estimation for online systems

Finite-time algorithms and applications

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OUTLINE

1. Non-A and finite time
2. Algebra
3. Homogeneity
4. Applications
5. Take-home message

1

NON-A AND FINITE-TIME

Team

Non-A

- Inria project-team (jointly with CNRS, EC Lille, Univ.Lille1)
- created July 2012 (initiated as a team in 2011)
- located in Lille (+ Nancy + Paris + Reims)
- 25 people from 11 countries (13 permanent, 6 PhD, 3 Post-Doc, 3 Eng.)

General positioning

Non-Asymptotic estimation for online systems

→ *a closer reading?*

General positioning

Non-Asymptotic estimation for **online systems**

→ *real-time signal, closed-loop control*

General positioning

Non-Asymptotic estimation for online systems

→ *finite-time algorithms*

(for specification, certification, separation...)

General positioning

Non-Asymptotic **estimation** for online systems

→ *infinitely many works!*

- parameters (identification)
- states (observation)
- derivatives (differentiation)
- inputs (left inversion)
- noisy data (filtering)

General positioning

General classification: (estimation techniques)

signal proc.

- output-based (model-free) estimation/differentiation
  - numerical differentiation
    finite diff. [Khan-Qu-Ramm...], Fourier transf. [Dou, Qian...], mollification [Hao, Murio...], regulariz. [Nakamura, Wang, Wei...], diff. by integration [Lanczos, Rangarajana, Wang,**Fliess-Mboup-Join**]
  - digital filtering
    [France: Chaplais, Diop... Abroad: AlAlaoui, Chen, Grizzle, Jackson, Lee, Mullis, Rader, Roberts...]

control/obs.

- model-based estimation
  [France: Besançon, Chitour, Gauthier, Glumineau-Moog-Plestan, Hamouri, Ibrir, Martin-Rouchon, Praly...
  Abroad: Allgöwer, Astolfi, Drakunov, Kazantsis-Kravaris, Khalil, Krener, Kupka, Li, Qian, Zeitz...]

Finite-time convergence:

- Hybrid/discontinuous ⇒ Nonsmooth/Sliding observers
  [Bartolini *et al.*, Brogliato *et al.*, Drakunov, Levant, Edwards-Spurgeon...]
- Continuous systems ⇒ Homogeneous observers
  [Andrieu-Praly-Astolfi, **Moulay - Perruquetti**, Shen-Xia...]

General positioning

Focus on two new standpoints for finite-time algorithms:

✓ Algebra — Fliess, Sira-Ramirez - ESAIM COCV 2003
An algebraic framework for linear identification

✓ Homogeneity — Perruquetti, Floquet, Moulay - IEEE TAC 2008
Finite-time observers: application to secure communication

usable for

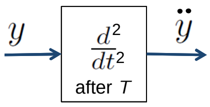
- numerical differentiation
- model-based observation
- identification or detection
- filtering...

2

ALGEBRAIC TECHNIQUES

An introduction with examples

Algebraic techniques: simple examples



→ double differentiation

Estimate $y(t) \to \frac{d^2y}{dt^2}$ over a sliding window with *small size* T:

$$y(t) = y(0) + y^{(1)}(0)t + \frac{1}{2}y^{(2)}(0)t^2,\ t \in [0, T]$$

$$\Rightarrow \hat{y}(s) = \frac{1}{s}y(0) + \frac{1}{s^2}y^{(1)}(0) + \frac{1}{s^3}y^{(2)}(0)$$

Apply the *annihilating operator:* $\frac{1}{s^3}\frac{d^2}{ds^2}s^2$:

$\times s^2$: $\Rightarrow s^2\hat{y} = sy(0) + y^{(1)}(0) + \frac{1}{s}y^{(2)}(0)$

$\frac{d^2}{ds^2}$: $\Rightarrow 2\hat{y} + 4s\frac{d\hat{y}}{ds} + s^2\frac{d^2\hat{y}}{ds^2} = \frac{2}{s^3}y^{(2)}(0)$

$\times\frac{1}{s^3}$: $\Rightarrow y^{(2)}(0) = \frac{5!}{T^5}\int_0^T \left(\tau^2 - 4(T-\tau)\tau + (T-\tau)^2\right) y(\tau)d\tau$

Algebraic techniques: simple examples

→ differentiation: a look on algebra

This derivative estimation involves the differential operator:

$$\Pi = \frac{1}{s^3}\frac{d^2}{ds^2}s^2 \in \mathbb{R}(s)\left[\frac{d}{ds}\right]$$

- defining a Weyl Algebra structure $\Rightarrow$ *canonical form:*

$$\Pi = \frac{1}{s}\frac{d^2}{ds^2} + \frac{4}{s^2}\frac{d}{ds} + \frac{2}{s^3}$$

right $\mathbb{R}(s)$ — left $\left[\frac{d}{ds}\right]$

Recall that the Weyl Algebra is non commutative: let $p = \frac{d}{ds}$ and $q = s\times$ then the *commutator* $[p,q] = pq - qp = 1$

R. Ushirobira, W. Perruquetti, M. Mboup, M. Fliess. *IFAC **SysId '12***, Brussels, **2012**

Algebraic techniques: simple examples

→ identification (simplest case!)

$$\dot{y}(t) = a y(t) + u(t) + \gamma_0$$

$$s\widehat{y}(s) - y(0) = a\widehat{y}(s) + \widehat{u}(s) + \frac{\gamma_0}{s}$$

$\times\, s$: $s^2\widehat{y}(s) - sy(0) = as\widehat{y}(s) + s\widehat{u}(s) + \gamma_0$

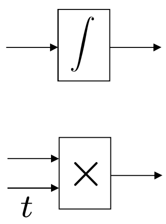
$\frac{d^2}{ds^2}$: $\left(-s\frac{d^2}{ds^2}\widehat{y}(s) - 2\frac{d}{ds}\widehat{y}(s)\right) a =$
$-2\widehat{y}(s) - 4s\frac{d}{ds}\widehat{y}(s) - s^2\frac{d^2}{ds^2}\widehat{y}(s) + 2\frac{d}{ds}\widehat{u}(s) + s\frac{d^2}{ds^2}\widehat{u}(s)$

$\times\, \frac{1}{s^3}$ ...

$\frac{1}{s^3}\frac{d^2}{ds^2}s$

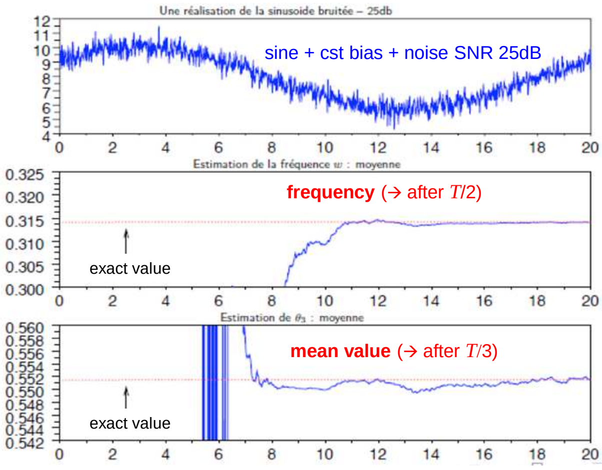
$$a = -\frac{\frac{1}{t}\int_0^1 (6\tau^2 - 6\tau + 1)\, y(t\tau)d\tau + \int_0^1 \tau(\tau-1)(2\tau-1)u(t\tau)d\tau}{\int_0^1 \tau(\tau-1)(2\tau-1)y(t\tau)d\tau}$$

→ toolbox



Algebraic techniques: simple examples

→ frequency estimation for noisy periodic signals



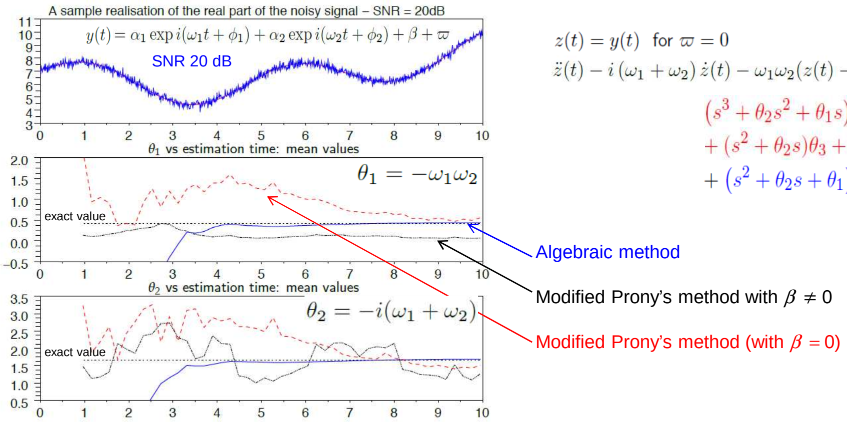
[Ushirobira, Perruquetti, Mboup, Fliess – GRETSI 2011]

Algebraic techniques: less simple examples

→ frequency estimation for noisy periodic signals

sum of two sine functions + meanvalue + noise

$$y(t) = \alpha_1 \exp i(\omega_1 t + \phi_1) + \alpha_2 \exp i(\omega_2 t + \phi_2) + \beta + \varpi$$



$z(t) = y(t)$ for $\varpi = 0$

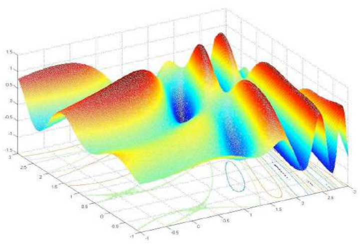
$\ddot{z}(t) - i(\omega_1 + \omega_2)\dot{z}(t) - \omega_1\omega_2(z(t) - \beta) = 0$

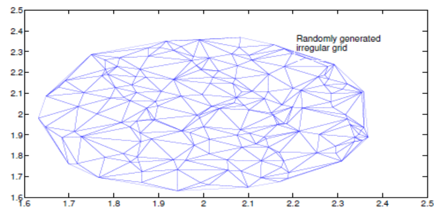
$$\left(s^3 + \theta_2 s^2 + \theta_1 s\right) Z(s) + (s^2 + \theta_2 s)\theta_3 + \theta_4 s + \left(s^2 + \theta_2 s + \theta_1\right)\theta_5 = 0$$

[Ushirobira, Perruquetti, Mboup, Fliess – IFAC SysId 2012]

Algebraic techniques: less simple examples

→ extension to differentiation of multi-variate signals





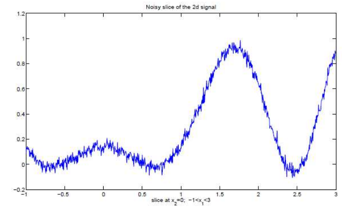


Figure 2: A slice of the noisy surface at $x_2 = 0$ and $-1 < x_1 < 3$, 25 dB

Irregular grids,
n-D noisy signal

(here, SNR 25dB)

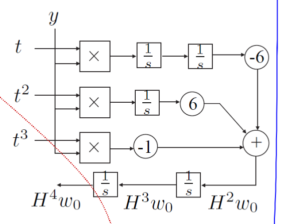
[Riachy, Mboup, Richard, JCAM2011]

Algebraic techniques: less simple examples

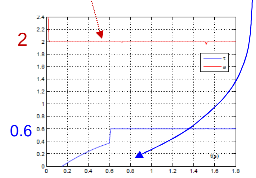
→ delay system (simulation, no noise)

$$\dot{y}(t) + ay(t) = y(0)\delta + \gamma_0 H + bu(t-\tau)$$

$y(0) = 0.3,\ a = 2,\ \tau = 0.6,$
$\gamma_0 = 2,\ b = 1,\ u_0 = 1.$



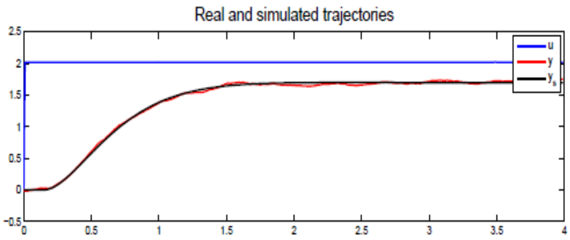
Simultaneous identification of a and τ

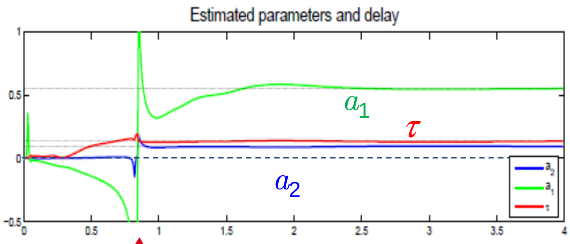


[L. Belkoura, J.P. Richard, M. Fliess. MTNS 2006, TDS 2006/2007]

Algebraic techniques: less simple examples

→ delay system (real, approx. 2nd order)





↑ ~ 0.8s

$$G_1(s) \approx \frac{0.84\, e^{-0.13\, s}}{0.09\, s^2 + 0.55\, s + 1}$$

$$= \frac{K\, e^{-\tau\, s}}{a_2 s^2 + a_1 s + 1}$$

Algebraic techniques: less simple examples

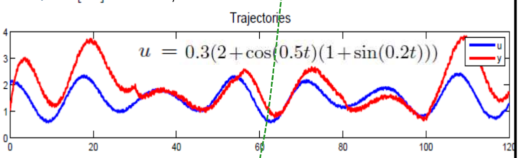
→ switched system

switch instants: $\tau_0 = 0 < \tau_1 < \tau_2 \ldots$

function: $b(t) = \tau_1 H(t - \tau_1)$

$$\dot{y} + a\, y = k\, u, \quad a(t) = \sum_i a_i\, \chi_{[\tau_i, \tau_{i+1}]}(t)$$

$\chi[X]$ = 0 or 1, characteristic function of the set X



$$\hat{a}_{|(t_0,t)} = \frac{k \int_{t_0}^{t} \theta u d\theta - t y + t_0 y(t_0) + \int_{t_0}^{t} y d\theta}{\int_{t_0}^{t} \theta y d\theta}$$

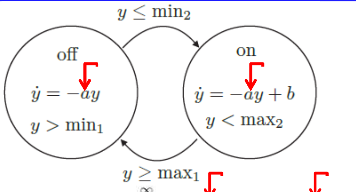
$(t_0, t) \not\ni \tau_i \quad i = 1, 2, \cdots$

filtering → $\hat{b}(t)$

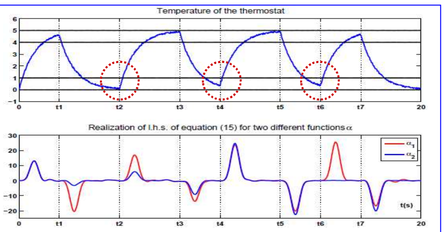
Algebraic techniques: less simple examples

→ impulsive systems
$$\sum_{j=0}^{n} a_j\, g_j^{(j)}(u,y) = \psi_0 + \sum_{j=1}^{\infty} b_j\, \delta(t-t_j).$$

The thermostat as a hybrid system



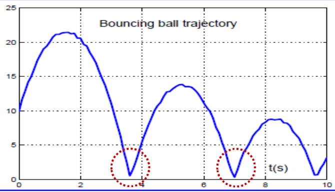
$$\ddot y + a\dot y = y_0\dot\delta_0 + b\sum_{j=0}^{\infty}\delta(t-t_{2j+1}) - \delta(t-t_{2j+2})$$



Bouncing ball / Rocking block

$$\frac{d^2y}{dt^2} = f(y) = \begin{cases} -\quad g & \text{(bouncing ball)} \\ \frac{1}{\alpha}\sin(\alpha\,(1-y\,\mathrm{sgn}(y))) & \text{(rocking block)} \end{cases}$$

$$\frac{dy}{dt}(t_k^+) = \gamma\frac{dy}{dt}(t_k^-), \quad k = 1, 2, \ldots,$$



Algebraic techniques:

→ some references – pdf on http://hal.archives-ouvertes.fr/

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  R. Ushirobira, W. Perruquetti, M. Mboup, M. Fliess. Algebraic parameter estimation of a biased sinusoidal waveform signal from noisy data. *IFAC **SysId '12***, Brussels, **2012**
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- *differentiation*
  M. Mboup, C. Join, M. Fliess. Numerical differentiation with annihilators in noisy environment. **Numerical Algorithms**, 50 (4), 439-467, **2009**.
  S. Riachy, M. Mboup, J.P. Richard. Multivariate numerical differentiation. **JCAM**, 236 (6): 1069-1089, **2011**
- *diagnosis*
  M. Fliess, C. Join, and H. Sira-Ramirez. Robust residual generation for linear fault diagnosis: an algebraic setting with examples. **Int. J. Control**, 77: 1223–1242, **2004**
- *delays & switches*
  L. Belkoura, J.P. Richard, M. Fliess. Parameters estimation of systems with delayed and structured entries. **Automatica**, 45 (5): 1117-1125, **2009**
  L. Belkoura. Identifiability and algebraic identification of time delay systems. In *Time Delay Systems: Methods, Appl. & New Trends*, **LNCIS** 423: 103-117, Springer, **2012**
- *impulses*
  L. Belkoura, K. IbnTaarit, T. Floquet, W. Perruquetti, Y. Orlov. Estimation problems for a class of impulsive systems. **Int. J. of Rob. & Nonlin. Control**, 21 (10): 1066-1079, **2011**

3

HOMOGENEITY AND FINITE-TIME

An introduction

Homogeneity and finite time

Again, very simple...

$$\dot{x} = g(x) = -\sqrt{|x|}\text{sign}(x)$$

For any $\lambda > 0$ and some d $\left(\text{here } d = \frac{1}{2}\right)$:

$$g(\lambda x) = \lambda^d g(x)$$

g is said to be homogeneous of degree d.

Note that:

- $g \in \mathcal{C}^0$ but is not Lipschitz $(0 < d < 1)$
- solutions $x(t)$ have a finite-time convergence

$$x_0 \geq 0 \rightarrow x(t) = (\sqrt{x_0} - \tfrac{t}{2})^2 \Rightarrow \boxed{T = 2\sqrt{x_0}}$$

Homogeneity and finite time

A bit more general...

Definition

For a function $f : \mathbb{R}^n \to \mathbb{R}^n$, if for any (positive) constant λ and all $x \in \mathbb{R}^n$

$$f(\lambda x) = \lambda^m f(x),$$

then the function f is called (positively) homogeneous with degree m.

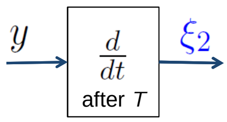
Theorem (Euler's theorem on homogeneous functions)

Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a C^1 homogeneous function of degree m, then

$$\frac{df(x)}{dx} x = mf(x).$$

Homogeneity and finite time

A homogeneous finite-time differentiator



$$\begin{cases} \dot{\xi}_1 = -k_1|\xi_1 - y|^{\alpha_1}\text{sign}(\xi_1 - y) + \xi_2, \\ \dot{\xi}_2 = -k_2|\xi_1 - y|^{\alpha_2}\text{sign}(\xi_1 - y) + \xi_3, \\ \quad\vdots \\ \dot{\xi}_n = -k_n|\xi_1 - y|^{\alpha_n}\text{sign}(\xi_1 - y). \end{cases}$$

with $0 < \alpha_i < 1$

Wilfrid Perruquetti, Thierry Floquet, and Emmanuel Moulay. Finite time observers : application to secure communication. *IEEE Transactions on Automatic Control*, 53(1) :356 360, 2008.

Advantages of homogeneous systems:

- local = global
- GAS = ISS (⇒ measurement noise robustness, additive disturbances compensation)
- finite-time stability / fixed-time stability

Homogeneity and finite time

Ongoing developments

1. How to generalize homogeneity to larger classes of systems? (ex: with delay)

   Denis Efimov and Wilfrid Perruquetti. Homogeneity for time-delay systems. In *IFAC WC 2011*, pages 1–6, Milano, Italy, August 2011.

2. "Finite-time" generally depends on the initial condition. Can one achieve T independently of x_0?
   $\rightarrow$ *Fixed*-time convergence

   Andrey Polyakov. Nonlinear feedback design for fixed-time stabilization of Linear Control Systems. *IEEE Transactions on Automatic Control*, 57(8):2106–2110, August 2012.

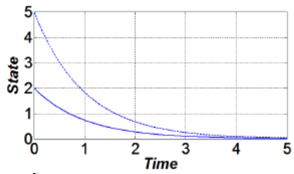
Homogeneity and finite time

Few words on fixed time stabilization

$$\begin{cases} \dot{x}(t) = u(t), \\ x(0) = x_0, \end{cases} \quad x, u \in \mathbb{R}.$$

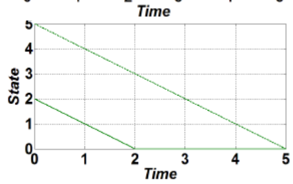
Asymptotic stability (*Lyapunov 1892*):

$$u(t) = -x(t)$$
$$x(t) = e^{-t}x_0 \rightarrow 0 \text{ if } t \rightarrow +\infty$$



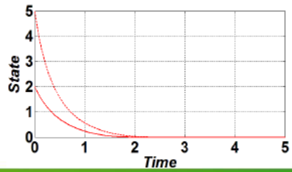
Finite-time stability (*Roxin 1966*):

$$u(t) = -\operatorname{sign}[x(t)]$$
$$x(t) = 0 \text{ for } t \geq \|x_0\|$$



Fixed-time stability (*Polyakov 2012*):

$$u(t) = -\left(|x(t)|^{1/2} + |x(t)|^{3/2}\right) \operatorname{sign}[x(t)]$$
$$x(t) = 0 \text{ for } t \geq \pi$$
independently of x_0



Homogeneity and finite time

Additionnal motivations for fixed-time stabilization

I. Robustness (Pervozvanski 1971)

$$\dot{x} = \lambda x + u$$

where $x \in \mathbb{R}$ - state, the number $\lambda \in \mathbb{R}$ is **unknown**, $u \in \mathbb{R}$ - control. For

$$u = -\mu x^3 \qquad \mu > 0$$

we have

if $\lambda > 0$ then $x \to \pm\sqrt{\lambda/\mu}$ as $t \to +\infty$ (practical stab.)

if $\lambda \leq 0$ then $x \to 0$ as $t \to +\infty$ (asympt. stab.)

II. Real-life applications for automobile engine control in GMC (Kolubin, Efimov et al. 2011)

$$u = -\alpha x - \beta \, \text{sign}[x] - \gamma x^3$$ « nonlinear PID-controller »

Homogeneity and finite time

Some references

- *homogeneity and finite-time observers*
  W. Perruquetti, T. Floquet, E. Moulay. Finite time observers: application to secure communication. **IEEE TAC**, 53 (1): 356-360, **2008**

  T. Menard, E. Moulay, W. Perruquetti. A global high-gain finite time observer. **IEEE TAC**, 55 (6): 1500-1506, **2010**
- *homogeneity and ISS*
  E. Bernuau, A. Polyakov, D. Efimov, W. Perruquetti. On ISS and iISS properties of homogeneous systems. ***ECC'13***, Zurich, July **2013**
- *homogeneity and finite-time control*
  E. Bernuau, W. Perruquetti, D. Efimov, E. Moulay. Finite-time output stabilization of the double integrator. ***CDC'12***, Maui, December **2012**
- *homogeneity and delay*
  D. Efimov, W. Perruquetti. Homogeneity for time-delay systems, ***IFAC'11***, Milano, August **2011**
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  A. Polyakov. Nonlinear feedback design for fixed-time stabilization of linear control systems. **IEEE TAC**, 57 (8): 2106-2110, **2012**

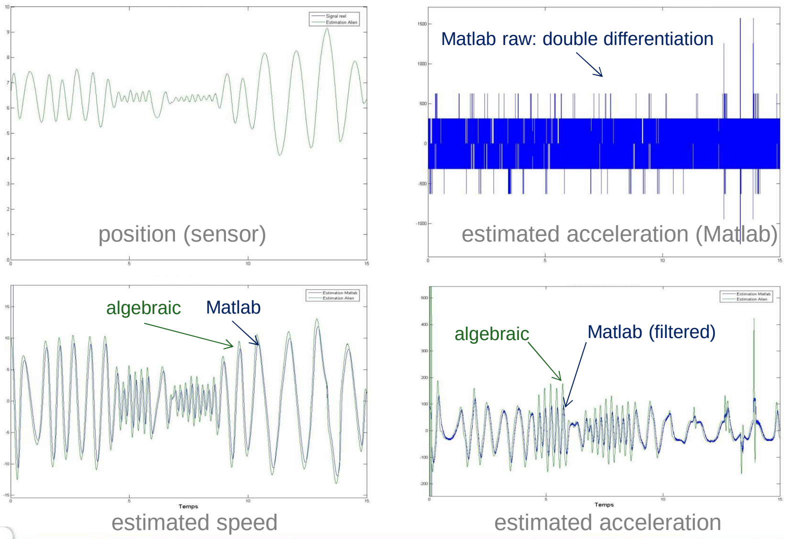
  S.A.. Kolyubin, D. Efimov, V.O. Nikiforov, A.A. Bobtsov. Two-channel adaptive hybrid control of the air-to-fuel ratio and torque of automobile engines. ***Automation and Remote Control***, 73 (11): 1794-1807, **2012**

4

APPLICATIONS

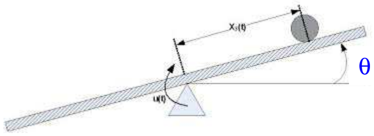
Applications

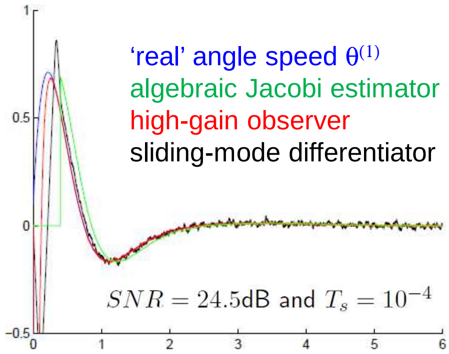
2nd order differentiation w.r.t. time (angle of a pendulum)

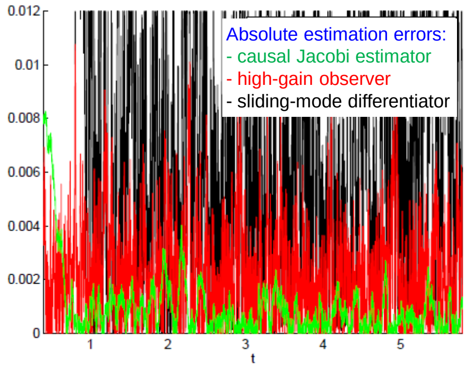


Applications

Ball and Beam system: 1rst order differentiation w.r.t. time

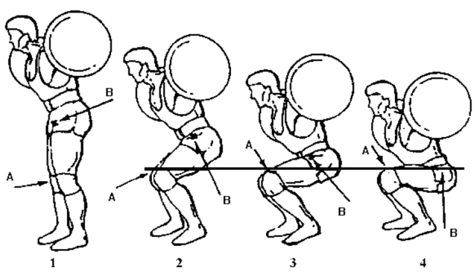




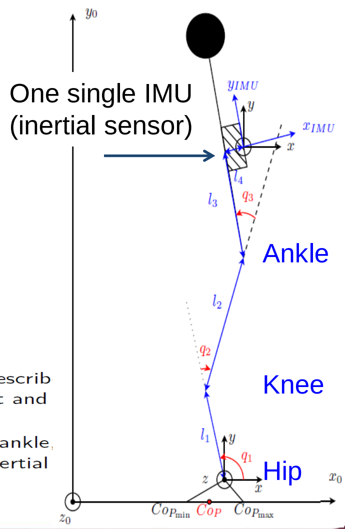


Applications

Human posture in the sagittal plane using... *accelerometer*



Squat exercise (periodic)

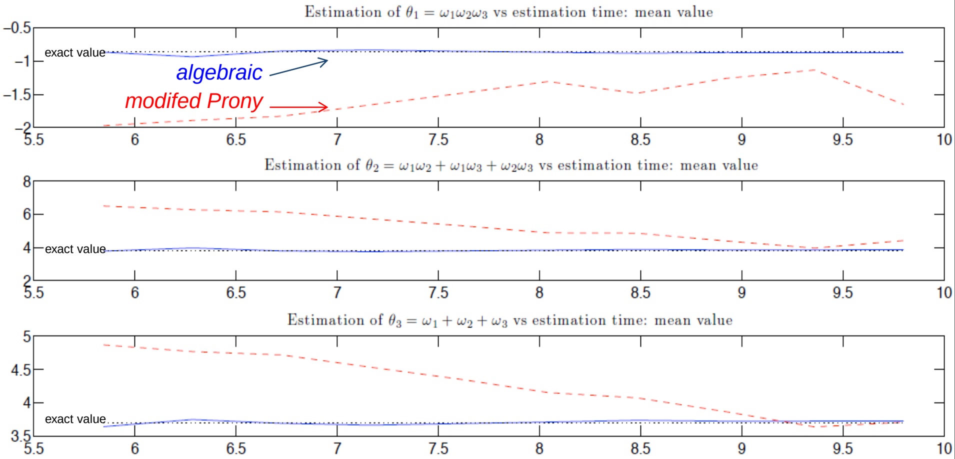


Goal: minimize the number of inertial sensor required to describ squat task, with the purpose of developing simple, low cost and daily life evaluation and rehabilitation tools.

Existing result: [Bonnet et al., 2012] proposes to estimate ankle, knee and hip joints during squat exercise using on single inertial sensor with an offline least-square estimation process.

Applications

Human posture in the sagittal plane using... *accelerometer*

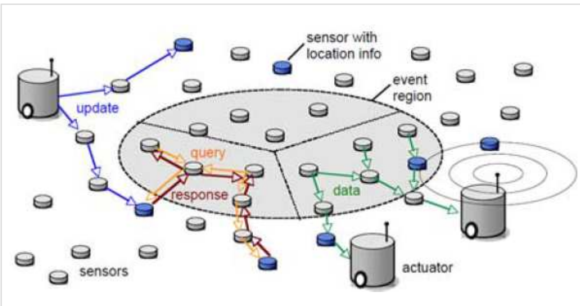


on-line algebraic estimation : Perruquetti et al. CDC 2012

Applications

General context: Wireless Sensor and Actuator Networks (WSAN)

deploy wireless sensor networks
by means of collaborating mobile robots

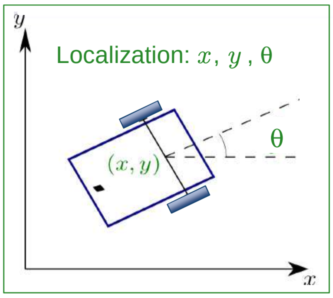


FIT:
Future
Internet
of Things
EquipEx
national program

with Inria *FUN*

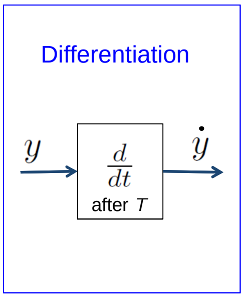
Hundreds of robots collaborating with thousands of sensors

Applications
WSAN: focus on localization

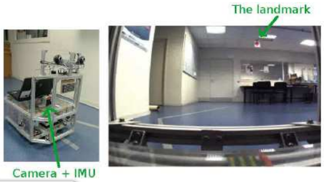


$$\tan\theta = \frac{\dot{y}}{\dot{x}}$$

flatness



→ *1 landmark + 1 target is enough*



H. Sert, A. Kokosy, and W. Perruquetti.
A single landmark based localization algorithm for non-holonomic mobile robots.
In *2011 IEEE International Conference on Robotics and Automation (ICRA)*, pages 293–298. IEEE, 2011.

H. Sert, W. Perruquetti, A. Kokosy, X. Jin, J. Palos, et al.
Localizability of unicycle mobiles robots : an algebraic point of view.
In *2012 IEEE International Conference on Intelligent Robots and Systems (IROS)*, 2012.

Applications
Some references

➢ *localization*

H. Sert, A. Kokosy, W. Perruquetti. A single landmark based localization algorithm for non-honolomic mobile robots. ***ICRA'11***, Shangai, China, May **2012**

H. Sert, W. Perruquetti, A. Kokosy, X. Jin, J. Palos. Localizability of unicycle mobile robots: an algebraic point of view. ***IROS'12***, Vilamoura, Portugal, October **2012**

➢ *ball and beam system*

D. Liu, O. Gibaru, W. Perruquetti. Error analysis of Jacobi derivative estimators for noisy signals, ***Numerical Algorithms***, 50 (4): 439-467, **2011**

➢ *position from accelerometer*

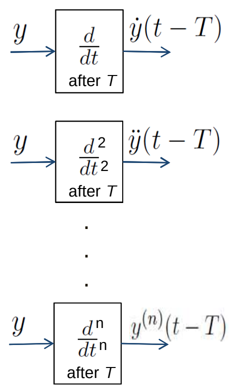
W. Perruquetti, V. Bonnet, M. Mboup, R. Ushirobira, P. Fraisse. An algebraic approach for human posture estimation in the sagittal plane using accelerometer noisy signal. **IEEE CDC**, Maui, USA, **2012**

5

TAKE-HOME MESSAGE

Take-home message

→ issue



Fast differentiation algorithms
- → delayed derivatives
- → with *small* delay
- → with *known* delay (algebra)

+ robust to noise
+ deterministic
+ feasible online →

Various fields are concerned:
- → signal processing
- → identification, observation
- → mode detection
- → control

→ toolbox