Adaptive Identification of Linear Time-Delay Systems

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Abstract

Synthesis of an adaptive parameter identifier is developed for linear dynamic systems with finitely many lumped delays in the state vector and control input. These systems are governed by linear functional differential equations with uncertain time-invariant parameters and delays. The state of the system is assumed to be available for measurements. Constructive necessary and sufficient conditions for the system parameters and delays to be identifiable are provided. Once the parameter identifiability is guaranteed the simultaneous on-line identification of the system parameters and delays is achieved by the adaptive identifier proposed. Theoretical results are supported by numerical simulation.

1 Introduction

Adaptive identification, also referred to as on-line identification, assumes construction of a model, parameters of which evolve in time and converge to the unknown parameters of the plant. Many aspects of the adaptive identifier design for lumped parameter systems are now well understood and due to the relative simplicity of implementation and some degree of robustness with respect to small perturbations of the plant dynamics, these identifiers found practical applications both by itself and as a part of an adaptive control system. There is already a large body of literature on this subject. In this regard, we refer to [1, 7, 12, 16, 22, 29] to name a few monographs.

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Recently [4, 6, 25], a general framework for Model Reference Adaptive Control (MRAC) of distributed parameter systems such as heat processes and mechanical oscillators with several unknown spatially varying parameters was proposed. It was demonstrated that it is in principle possible to simultaneously identify all the plant parameters both in the case of heat processes and that of mechanical oscillators.

In comparison to the lumped and distributed parameter systems, adaptive control of delay systems is not well-developed and has only recently been studied in [5, 11, 32] within the framework of methods which do not seek to identify the underlying uncertain process, but which seek its control only. In turn, the first works [21, 31] on the identification of time-delay systems have demonstrated complexity of the problem, particularly, the identifiability of a delay system was shown to place a restrictive condition on the structure of the system. This condition was defined through the characteristic matrix of the functional differential equation of the plant whereas no indication was given on how to attain this condition using some accessible inputs.

In our recent works [3, 23, 24], constructively verifiable and enforceable conditions for a linear delay system to be, in principle, identifiable have been given in terms of the well-known weak controllability property and nonsmooth input signals.

Once the identifiablity of the system is guaranteed, the task of the plant identification can be carried out by a proper adaptive identifier design which effectively utilizes the prior knowledge of the plant structure through its inclusion into the reference model, whose parameters evolve in time and converge to the unknown parameters of the plant.

In the present paper, we extend the Lyapunov-Razumikhin redesign techniques, developed in [2] for a scalar delay system with the only delay in the state, to general linear dynamic processes with delayed states and control inputs. These processes, also referred to as linear delay differential systems, are governed by linear vector functional differential equations. As in the finite-dimensional case, it is assumed that the structure of the underlying system is known and only system parameters and delays are unknown. The state of the system is also assumed to be available for measurement.

The adaptive identifier constructed is given in terms of functional differential equations, describing the evolution of the state estimate and the parameter estimate. The structure of the identifier is inspired from the Lyapunov-Razumikhin redesign techniques and has what is called the series-parallel configuration. However, in contrast to the finite-dimensional treatment (cf. that of [12, 22]), it becomes impossible to repeat the structure of the identified system while system delays remain unknown. By virtue of this, the identifier needs to involve a number of delays larger than that of the system, theoretically distributed delays, so that some estimated parameters correspond to a priori unknown fictitious delays.

The justification of the convergence of the tunable parameters to their nominal values is based on an extension to delay systems of the Krasovskii - La Salle's invariance principle [15, 17], given in [8]. For the coupled plant/identifier system, a Lyapunov-Razumikhin function with a nonpositive time derivative along the solutions of the system is constructed. This time derivative takes zero values on a certain manifold in the state space. Applying a periodic sufficiently nonsmooth input signal guarantees the absence of the nontrivial trajectories on the manifold, thereby yielding the convergence of the tunable parameters to their nominal values. In particular, the fictitious delays are carried out by the vanishing tunable parameters.

Simulation of the adaptive identifier demonstrates favorable robustness properties against small uncertainties on the system delays. It comes from simulation, too, that the convergence of the tunable parameters remains in force even if a certain smooth sufficiently rich input signal is applied to the system.

The outline of the paper is as follows. Section 2 reviews some mathematical preliminaries for linear delay differential systems. The adaptive algorithm of identification of the system parameters and delays is developed in Section 3. Section 4 supports theoretical results by numerical simulations. Section 5 presents the conclusions.

1.1 Notation

Notation is fairly standard. $\mathbf{R}^n$ and $\mathbf{C}^n$ are n-dimensional Euclidean spaces, real- and complex-valued, respectively. $C^l(\mathbf{R}^n)$ is the Banach space of $l-$times continuously differentiable n-vector functions on $(-\infty, \infty)$, $l \geq 0$, the integer; $C^\infty(\mathbf{R}^n) = \bigcap_{l\geq 0} C^l(\mathbf{R}^n)$. $L_2(a, b; \mathbf{R}^n)$ is the Hilbert space of n-vector functions, square integrable on $[a, b]$. S^* stands for the adjoint operator of S.

2 Preliminaries

In this section, we address some basic facts for linear time-delay systems governed by functional differential equations of the form:

$$\dot{x}(t) = \Sigma_{i=0}^{r}[A_i x(t-\tau_i) + B_i u(t-\tau_i)], \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is a state vector, $u(t) \in \mathbf{R}^p$ is a piece-wise continuous control input, $0 = \tau_0 < \tau_1 < \ldots < \tau_r$ are time delays, $A_i \in \mathbf{R}^{n\times n}, B_i \in \mathbf{R}^{n\times p}$, $i = 0, \ldots, r$ are matrix parameters. Since some matrix parameters might be zero, without loss of generality we have assumed that system (1) has the same state and input delays.

The above state equation is subject to the initial condition

$$x(-\theta) = \varphi(\theta),\ 0 \leq \theta \leq \tau_r, \tag{2}$$

which is assumed to be of class C^∞. It is well-known (see, e.g., [9, 14]) that given $\varphi(\theta)$ and $U(t) = \Sigma_{i=0}^{r} B_i u(t-\tau_i)$, there exists a unique solution of (1) subject to (2). This solution is given by

$$x_t = S(t)\varphi + \int_0^t S(t-s)U_s ds \tag{3}$$

where $U_s(\theta) = U(s-\theta)$ is a function of the argument $\theta \in [0, \tau_r]$; $S(t)$ is a C_0-semigroup on the Hilbert space $\mathbf{R}^n \times L_2(0, \tau_r; \mathbf{R}^n)$. If the linear system (1) is asymptotically stable under $u \equiv 0$ then the semigroup $S(t)$ is exponentially stable. Just in case the Lyapunov-Krasovskii functional

$$V(x) = < Wx, x >,$$

defined by means of the inner product $< \cdot, \cdot >$ in $R^n \times L_2(0, \tau_r; R^n)$ and the operator

$$W = \int_0^\infty S^*(t)S(t)dt, \tag{4}$$

has the negative definite time derivative

$$\dot{V}(t) = - < x_t, x_t >$$

along the trajectories of the homogeneous ($u \equiv 0$) system (1). In addition, if the function $u(t)$ is periodic, the steady-state behavior of the system is a periodic orbit of (1) given by

$$z_t = \int_{-\infty}^t S(t-s)U_s ds. \tag{5}$$

Along with the delay differential system (1), we shall also consider its representation

$$\begin{aligned} \dot{x}(t) &= A(\lambda)x(t) + B(\lambda)u(t), & (6)\\ y(t) &= C(\lambda)x(t) & (7) \end{aligned}$$

over the ring $\mathbf{R}[\lambda]$ of polynomials in a vector variable $\lambda = (\lambda_1, ..., \lambda_k)^T$ with real coefficients (see [30] for details). Hereinafter k is the maximal number of non-commensurable delay units $\eta_1, ..., \eta_k,\ i = 1, \ldots, k$ in (1) and $\lambda_1, ..., \lambda_k$ are fixed delay operators $(\lambda_i x)(t) = x(t - \eta_i)$ of these non-commensurable units.

Recall that units $\eta_1, ..., \eta_k$ are commensurable if there exist integers $\alpha_1, ...,$ α_k such that $\sum_{i=1}^k \alpha_i \eta_i = 0$ and $\sum_{i=1}^k \alpha_i^2 \neq 0$ and, respectively, $\eta_1, ..., \eta_k$ are not commensurable if $\sum_{i=1}^k \alpha_i \eta_i = 0$ with integers $\alpha_1, ..., \alpha_k$ implies that $\sum_{i=1}^k \alpha_i^2 = 0$.

In representation (6), (7) $x(\cdot)$, $u(\cdot)$, and $y(\cdot)$ take values in the free finitely generated modules $\mathbf{R}^n[\lambda]$, $\mathbf{R}^p[\lambda]$, and $\mathbf{R}^q[\lambda]$ of $n \times 1$, $p \times 1$, and $q \times 1$ $\mathbf{R}[\lambda]$-matrices (i.e., matrices whose entries are polynomials in λ), respectively.

The general weak controllability concept is similar to that introduced in [20] for particular commensurable delay systems.

Definition 1 System (1) is said to be weakly controllable or controllable over the field $\mathbf{R}(\lambda)$ of rational functions in the vector variable λ with real coefficients if there exists a rational feedback control law

$$u(x(t), x(t-\tau_1), x(t+\tau_1), \ldots, x(t-\tau_r), x(t+\tau_r), x(t-2\tau_1), x(t+2\tau_1), \ldots)$$

driving the system from an arbitrary initial state $x_0 \in \mathbf{R}^n[\lambda]$ to an arbitrary element of the module $\mathbf{R}^n[\lambda]$.

In matrix terms system (1) is weakly controllable iff

$$rank[B(\lambda) \mid A(\lambda)B(\lambda) \mid \ldots \mid A^{n-1}(\lambda)B(\lambda)] = n \tag{8}$$

over $\mathbf{R}[\lambda]$, or equivalently

$$rank\ \left[B(z) \mid A(z)B(z) \mid \ldots \mid A^{n-1}(z)B(z)\right] = n \tag{9}$$

for some $z \in \mathbf{C}^k$.

The concept of the system identifiablity is based on the comparison of system (1) and its reference model

$$\dot{\hat{x}}(t) = \Sigma_{i=0}^{\hat{r}}[\hat{A}_i\hat{x}(t-\hat{\tau}_i) + \hat{B}_i u(t-\hat{\tau}_i)], \tag{10}$$

in which the system parameters A_i, B_i, C_i and delays $\tau_i,\ i = 0, \ldots, r$ are replaced by $\hat{A}_i, \hat{B}_i$, and $\hat{\tau}_i,\ i = 0, \ldots, \hat{r}$, respectively. Throughout, the reference model (10) is subject to the initial condition

$$\hat{x}(-\theta) = \hat{\varphi}(\theta),\ 0 \le \theta \le \tau_{\hat{r}}, \tag{11}$$

of class C^∞, and symbol $\hat{(.)}$ stands for the corresponding model variables.

Definition 1 *System (1) is said to be identifiable if there exists a control input* $u(t)$ *such that the identity* $x(t) \equiv \hat{x}(t)$ *results in*

$$r = \hat{r},\ \tau_i = \hat{\tau}_i,\ A_i = \hat{A}_i,\ B_i = \hat{B}_i\ for\ i = 0, \ldots, r,$$

regardless of a choice of the initial functions $\varphi(\theta), \hat{\varphi}(\theta)$*. In that case the identifiability is said to be enforced by the control input* $u(t)$*.*

The time-delay system (1) is in principle identifiable if and only if this system is weakly controllable. Identifiability of the system parameters and delays can then be enforced by applying to the system a sufficiently nonsmooth signal. These signals are constructively introduced by imposing the state of the system and the system input to have different smoothness properties (rather than different frequencies as is typically the case in the finite-dimensional theory).

Definition 2 *An input signal $u(t)$ is piece-wise regular on $[-\tau_r, \infty)$ iff there exists an increasing sequence $\{t_j\}_{j=1}^{\infty}$ of time instants $-\tau_r = t_1 < t_2 < \dots$ with $\lim_{j\to\infty} t_j = \infty$ such that $u(t)$ is of class $C^{\infty}(t_j, t_{j+1})$ on each subinterval $(t_j, t_{j+1}),\ j = 1, 2, \dots$ and its l-th order derivative $u^{(l)}(t)$ has finite unilateral limits at the points t_j for all $l = 0, 1, \dots$, i.e., $u(t+) = \lim_{t\to t_j+} u^{(l)}(t) < \infty$ and $u(t-) = \lim_{t\to t_j-} u^{(l)}(t) < \infty$.*

Definition 3 *A piece-wise regular input signal $u(t)$ is said to be sufficiently discontinuous for system (1) and its model (10) if the following conditions are satisfied:*

1. *The set D of discontinuity points t_j, $j = 1, 2, ...$ of $u(t)$ contains a subset $D_0 \subset D$ which consists of at least $(p + k + \hat{k})$ non-commensurable points.*
2. *The jumps $\sigma_t = u(t+) - u(t-)$ at arbitrary p points $t_{\mu_i} \in D_0$, $i = 1, 2, ..., p$ form a matrix of the full rank, i.e.,*

$$rank \left[\sigma_{t_{\mu_1}}, \dots, \sigma_{t_{\mu_p}}\right] = p. \tag{12}$$

For ease of reference, the above conditions 1 and 2 are further referred to as the non-commensurability condition and the rank condition, respectively. Apparently, the rank condition does not relate to the plant equations whereas the non-commensurability condition explicitly utilizes the maximal number k of non-commensurable system delay units (as well as that $\hat{k}$ of the model) that can be unrealistic from a practical standpoint. To avoid this drawback one can modify the non-commensurability condition by requiring the set D_0 to consist of countable number of discontinuity points of $u(t)$ so that both the non-commensurability condition and the rank condition can be verified independently of any underlying system. This feature allows us to subsequently ignore the underlying system and its model while constructing a sufficiently rich control input.

By definition, in order to construct a sufficiently discontinuous input signal one should, firstly, select a sufficient number of non-commensurable discontinuity points $t_j,\ j = 1, 2, \dots$ (in particular, p points for non-delay systems when $k = 0$), secondly, construct corresponding input jumps σ_j which satisfy the rank condition, and thirdly, impose a smooth behavior on the input signal between the discontinuity points, while the prespesified input jumps are provided at these points.

The ability of construction of sufficiently many vectors $\sigma_j \in \mathbf{R}^p\ j = 1, 2, \dots$, which satisfy the rank condition, is demonstrated as follows. Construction of p such vectors $\sigma_j\ j = 1, \dots, p$, as a matter of fact, consists in specifying a basis $\Omega = \{e_i\}_{i=1}^{p}$ in the Euclidean space $\mathbf{R}^p$. Since the union $\Sigma_p = \cup \mathbf{R}_{\sigma_{\mu_1}, \dots, \mu_{p-1}} \subset \mathbf{R}^p$ of all $(p-1)$-dimensional subspaces $\mathbf{R}_{\sigma_{\mu_1}, \dots, \mu_{p-1}} \subset \mathbf{R}^p$ spanned by arbitrary $p - 1$ basis vectors $\sigma_{\mu_i} \in \Omega,\ i = 1, \dots, p - 1$ is of zero measure, it is always possible to find a vector $\sigma_{p+1} \in \mathbf{R}^p$ which does not belong to Σ_p. It follows that $p+1$ vectors σ_j $j = 1, \ldots, p+1$, thus constructed, satisfy the rank condition as well. An arbitrarily large number of vectors, satisfying the rank condition, is then obtained by iteration.

It is important from a practical standpoint to extend Definition 4 to the case when system (1) is enforced by a continuous input signal, in particular, when the infinite slop of a sufficiently discontinuous input signal is approximated by a finite one.

Definition 4 *A piece-wise regular input signal $u(t)$ of class $C^l, l \geq 1$ is said to be sufficiently nonsmooth if its l-th order time derivative $u^{(l)}(t)$ is a sufficiently discontinuous input signal.*

For the later use, the following result is extracted from [3, 23, 24].

Theorem 1 *The time-delay system (1) is identifiable if and only if it is weakly controllable. Moreover, if (1) is weakly controllable then the identifiability can be enforced by any sufficiently nonsmooth control input $u(t)$.*

Restricted to non-delay systems, the above result is straightforwardly related to the classical results on parameter identifiability. Indeed, the weak controllability of a non-delay linear dynamic system is nothing else than its controllability whereas sufficiently nonsmooth input signals, expanded into appropriate Fourier series, are known [19] to have enough frequencies and hence be sufficiently rich.

Furthermore, in analogy to the non-delay case [19], one could hopefully introduce a sufficiently rich input signal which would be independent, in a certain sense, on the state vector in the time-delay case as well. In the sequel we demonstrate, with the help of simulations, that all of the system parameters and delays are identifiable even if a smooth sufficiently rich input signal is applied to the time-delay system.

3 Adaptive Identifier Design

Once the identifiability of system (1) is guaranteed, the task of the identification of the unknown matrix parameters $A_i, B_i, i = 0, \ldots, r$ and delays $\tau_j,\ j = 1, \ldots, r$ can be carried out by the adaptive identifier proposed below. Throughout this section, the unforced delay system (1) is assumed to be asymptotically stable under $u \equiv 0$. For a moment the time delay values $\tau_j,\ j = 1, \ldots, r$ are assumed to be known a priori.

3.1 Identifier Synthesis with the Apriori Known Delays

Let $G \in R^{n \times n}$ be a Hurwitz matrix and let $P, Q \in R^{n \times n}$ be positive definite symmetric matrices such that

$$G^T P + PG = -Q.$$

The following identification law is then proposed in order to identify the matrix parameters $A_i,\ B_i,\ i = 0, \dots, r$:

$$\begin{aligned} \dot{\hat{x}}(t) = \Sigma_{i=0}^{r}[\hat{A}_i x(t-\tau_i) + \hat{B}_i u(t-\tau_i)] - G\Delta x(t),\ t \geq 0, \\ \hat{x}(-\theta) = \hat{\varphi}(\theta),\ 0 \leq \theta \leq \tau_r, \end{aligned} \tag{13}$$

$$\begin{aligned} \dot{\hat{A}}_i = F_i P \Delta x(t) x^T(t-\tau_i),\ \hat{A}_i(0) = \hat{A}_i^0, \\ \dot{\hat{B}}_i = \Phi_i P \Delta x(t) u^T(t-\tau_i),\ \hat{B}_i(0) = \hat{B}_i^0 \end{aligned} \tag{14}$$

where $\hat{A}_i^0 \in \mathbf{R}^{n\times n}, \hat{B}_i^0 \in \mathbf{R}^{n\times m}$, the state error $\Delta x(t) = x(t) - \hat{x}(t)$, the initial function $\hat{\varphi}(\theta)$ is of class C^∞, the adaptation gain matrices F_i, Φ_i are positive definite and of appropriate dimensions. This law is derived from Lyapunov redesign technique arguments [12, 22], which are now extended to time-delay systems.

The identification law, as shown below, ensures the convergence

$$\lim_{t\to\infty} \Delta x(t) = 0,\ \lim_{t\to\infty} \Delta A_i(t) = 0,\ \lim_{t\to\infty} \Delta B_i(t) = 0,\ i = 0, 1, \dots, r \tag{15}$$

of the state error $\Delta x(t)$ and the parameter errors $\Delta A_i(t) = A_i - \hat{A}_i(t)$,$\Delta B_i(t) = B_i - \hat{B}_i(t)$ for arbitrary adaptation gains, initial conditions and periodic input signals, enforcing the identifiability of the system.

Theorem 2 *Let the following conditions be satisfied:*

1. *the time delay values $\tau_j,\ j = 1, \dots, r$ are known a priori;*
2. *system (1) is identifiable and its unforced version under $u = 0$ is asymptotically stable;*
3. *the control input $u(t)$ enforces the system identifiability and it is periodic.*

Then the limiting relations (15) hold with the adaptive identification law (13) and parameters $\hat{A}_i(t)$ and $\hat{B}_i(t),\ i = 0, \dots, r$ tuned as (14).

Proof of Theorem 2 Let us represent the over-all system (1), (13), (14) in terms of the output-parameter errors:

$$\begin{aligned} \dot{e}(t) = \Sigma_{i=0}^{r} A_i e(t-\tau_i), \\ \Delta\dot{x}(t) = \Sigma_{i=0}^{r}\{\Delta A_i[e(t-\tau_i) + z(t-\tau_i)] + \Delta B_i u(t-\tau_i)\} + G\Delta x(t), \\ \Delta\dot{A}_i(t) = -F_i P \Delta x(t)[e(t-\tau_i) + z(t-\tau_i)]^T, \\ \Delta\dot{B}_i = -\Phi_i P \Delta x(t) u^T(t-\tau_i),\ i = 0, \dots, r \end{aligned} \tag{16}$$

where $e(t) = x(t) - z(t)$ is the state deviation with respect to the periodic orbit $z(t)$ of (1) which is given by (5).

Now let us prove that (16) is globally asymptotically stable. For this purpose, we introduce the Lyapunov functional

$$V(e, \Delta x, \Delta A_0, \dots, \Delta B_l) =< We, e > +\Delta x^T P \Delta x + \Sigma_{i=0}^r [tr(\Delta A_i^T F_i^{-1} \Delta A_i) + tr(\Delta B_i^T \Phi_i^{-1} \Delta B_i)] \quad (17)$$

where tr denotes the trace of a matrix, W is defined by relation (4) with the exponentially stable semigroup $S(t)$ of the former equation of (16). The computation of the time-derivative of the Lyapunov functional V along the trajectories of (16) yields

$$\dot{V}(t) = - < e_t, e_t > -\Delta x^T(t) Q \Delta x(t).$$

Although this time-derivative is only negative semidefinite, global asymptotic stability of (16), the right-hand side of which is apparently time-periodic, is established by invoking the extension of the Krasovskii - LaSalle's invariance principle to delay systems. Such an extension, given in [8] for time-invariant delay systems, admits a straightforward generalization to the periodic case, similar to the one obtained in [28] for non-delay systems.

According to the invariance principle, there must be a convergence of the trajectories of (16) to the largest invariant subset of the set of solutions of (16) for which $\dot{V}(t) \equiv 0$, or equivalently

$$e(t) \equiv 0, \;\; \Delta x(t) \equiv 0. \quad (18)$$

The manifold of (18), however, does not contain nontrivial trajectories of (16). Indeed, if (18) is in force, then taking into account (14), it follows that

$$\dot{\hat{A}}_i(t) = 0, \;\; \dot{\hat{B}}_i(t) = 0, \;\; i = 0, \dots, r. \quad (19)$$

In turn, coupled to (16), relations (18), (19) result in

$$\Sigma_{i=0}^r [\Delta A_i \, x(t - \tau_i) + \Delta B_i u(t - \tau_i)] \equiv 0$$

which by identifiability of (1) (cf. Definition 1) implies that

$$\Delta A_i = 0, \;\; \Delta B_i = 0, \;\; i = 0, \dots, r.$$

Thus, by applying the invariance principle system (16) is globally asymptotically stable and the required identifier convergence (15) is concluded. Theorem 2 is proven.

Remark 1 *Since the Krasovskii - LaSalle's invariance principle is not applicable to a general nonautonomous system [13] the periodicity condition imposed by Theorem 2 on the input signal can not be omitted. However, the assertion of Theorem 2 remains true if a periodic input signal $u(t)$, enforcing the system identifiability, is replaced by a non-periodic one which saves the invariance principle (examples are almost-periodic signals and asymptotically autonomous signals [13]).*

Remark 2 *If some plant parameters are known a priori then the corresponding identifier equations can be omitted.*

The above parameter identifier is subsequently shown, by a way of simulation, to have a favorable robustness property against small uncertainties on the time delays. This property and Theorem 2, coupled together, allow one to simultaneously identify, with a certain precision, all the matrix parameters and the delays of the system.

3.2 Identifier Synthesis with the Apriori Unknown Delays

The following identifier design procedure is proposed when knowledge of the delay values is no longer available.

On the first step system (1) is modeled as an uncertain system with a priori known delays $\hat{\tau}_i,\ i = 1, \ldots, \hat{r}$ closely located to each other:

$$|\hat{\tau}_i - \hat{\tau}_{i-1}| < \epsilon,\ i = 1, \ldots, \hat{r} \tag{20}$$

where $\epsilon > 0$ is an identification precision. Since the earlier-obtained identifiability conditions do not depend on the commensurability of the delay values the model delays $\hat{\tau}_1, \ldots, \hat{\tau}_{\hat{r}}$ can uniformly be distributed over the time interval $[0, h]$ with $h \geq \tau_r$ being an upper estimate of the system delays. Certainly, in order to ensure the prescribed precision, the number $\hat{r}$ of the model delays is selected large enough so that some model matrix parameters (unknown, however, a priori) correspond to fictitious delays.

After that a parameter identifier (13), (14), being constructed for the aforementioned model, completes the design procedure. Indeed, by Theorem 2 the identifier parameters converge to the corresponding model parameters and due to the robustness of the identifier against small uncertainties on the delay values these parameters approximate those of the system whereas vanishing identifier parameters (i.e., those converging to zero as $t \to \infty$) carry out fictitious delays.

In fact, the above procedure is nothing else than a discrete realization of the following adaptive identifier with a distributed delay:

$$\dot{\tilde{x}}(t) = \int_0^h [\tilde{\xi}_A(t,\theta)x(t-\theta) + \tilde{\xi}_B(t,\theta)u(t-\theta)]d\theta - G[x(t) - \tilde{x}(t)],\ t \geq 0,$$

$$\tilde{x}(-\theta) = \tilde{\varphi}(\theta),\ 0 \leq \theta \leq h, \tag{21}$$

$$\dot{\tilde{\xi}}_A(t,\theta) = F(\theta)P[x(t) - \tilde{x}(t)]x^T(t-\theta),\ \tilde{\xi}_A(0,\theta) = \tilde{\xi}_A^0(\theta),$$

$$\dot{\tilde{\xi}}_B(t,\theta) = \Phi(\theta)P[x(t) - \tilde{x}(t)]u^T(t-\theta),\ d\tilde{\xi}_B(0,\theta) = \tilde{\xi}_B^0(\theta) \tag{22}$$

where the initial conditions $\tilde{\varphi}(\theta), d\tilde{\xi}_A^0(\theta), d\tilde{\xi}_B^0(\theta)$ are smooth functions of appropriate dimensions, the adaptation gain matrix functions $F(\theta), \Phi(\theta)$ are continuous, everywhere positive definite, and of appropriate dimensions, the matrices P and G are the same as before. In order to make the sense of the tuned functions

$\tilde{\xi}_A(t,\theta), \tilde{\xi}_B(t,\theta)$ clear one should represent the state equation (1) in a similar form

$$\dot{x}(t) = \int_0^h [\xi_A(\theta)x(t-\theta) + \xi_B(\theta)u(t-\theta)]d\theta,\ t \geq 0, \tag{23}$$

where the distributions $\xi_A(\theta) = \Sigma_{i=0}^{r} A_i \delta(\theta - \tau_i), \xi_B(\theta) = \Sigma_{i=0}^{r} B_i \delta(\theta - \tau_i)$ have a one-to-one relation to the system parameters and delays, $\delta(t-\tau)$ is a Dirak function, atomized at $t = \tau$.

The identification law (21), (22) is shown to guarantee the state error convergence

$$\lim_{t\to\infty} \|\tilde{x}(t) - x(t)\| = 0 \tag{24}$$

as well as the weak* convergence

$$* - \lim_{t\to\infty} \tilde{\xi}_A(t,\theta) = \xi_A(\theta),\ * - \lim_{t\to\infty} \tilde{\xi}_B(t,\theta) = \xi_B(\theta) \tag{25}$$

of the tuned distributions to the nominal ones for arbitrary adaptation gains, initial conditions and periodic input signals, enforcing the identifiability of the system. Since the weak* convergence (25) ensures that

$$\lim_{t\to\infty} \tilde{\xi}_A(t,\theta) = 0,\ for\ all\ \theta \neq \tau_i,\ i = 0, \ldots, r \tag{26}$$

and

$$\lim_{t\to\infty} \int_{\tau_i-\varepsilon}^{\tau_i+\varepsilon} \tilde{\xi}_A(t,\theta)d\theta = A_i,\ \lim_{t\to\infty} \int_{\tau_i-\varepsilon}^{\tau_i+\varepsilon} \tilde{\xi}_B(t,\theta)d\theta = B_i, \imath = 0, \ldots, r \tag{27}$$

for all intervals $(\tau_i - \varepsilon, \tau_i + \varepsilon), \varepsilon > 0$ with the only delay value τ_i inside, the identifier (21), (22) yields the nominal parameters and delay values as $t \to \infty$.

Summarizing, the following result is obtained.

Theorem 3 *Let the time delay values $\tau_j,\ j = 1, \ldots, r$ be unknown a priori whereas Conditions 2 and 3 of Theorem 2 be satisfied. Then the limiting relations (24), (25) hold with the adaptive identification law (21) and distributions $\tilde{\xi}_A(t,\theta), \tilde{\xi}_B(t,\theta)$ tuned as (22).*

Remarks 1 and 2 apply here for the identification of unknown delays as well.

4 Simulation Results

Performance issues of the adaptive identifier were studied by simulation of a scalar system

$$\dot{x}(t) = a_0x(t) + a_1x(t-\tau_1) + b_0u(t),\ x, u \in \mathbf{R} \tag{28}$$

with a single delay in the state. Apparently system (28) is weakly controllable if and only if $b_0 \neq 0$. Thus by Theorem 1 system (28) with $b_0 \neq 0$ turns out to be identifiable and its identifiability is enforced by a sufficiently nonsmooth control input. If, in addition, the unforced system (28) subject to $u = 0$ is asymptotically stable ($a_0 < 0,\ a_0 + |a_1| < 0$) then the parameters of the system can be identified via the proper adaptive identifier design.

Applied to system (28), identifier (21) (22) is discretized as follows:

$$\dot{\hat{x}}(t) = \Sigma_{i=0}^{\hat{r}}[\hat{a}_i x(t - \hat{\tau}_i) + \hat{b}_i u(t - \hat{\tau}_i)] + a\Delta x(t),\ t \geq 0, \tag{29}$$

$$\dot{\hat{a}}_i = \beta_i \Delta x(t) x^T(t - \hat{\tau}_i),$$

$$\dot{\hat{b}}_i = \gamma_i \Delta x(t) u^T(t - \hat{\tau}_i),\ i = 0, \ldots, \hat{r} \tag{30}$$

where the identifier delays satisfy the prescribed precision (20), $a > 0,\ \beta_i > 0,\ \gamma_i > 0,\ i = 0, \ldots, \hat{r}$ are adaptation parameters. If the structure of the system as well as the state delay τ_1 would be known a priori then the identifier equations could be simplified as follows:

$$\dot{\hat{x}}(t) = \hat{a}_0 x(t) + \hat{a}_1 x(t - \tau_1) + \hat{b}_0 u(t) + a\Delta x(t),\ t \geq 0, \tag{31}$$

$$\dot{\hat{a}}_0 = \beta_0 \Delta x(t) x^T(t),\ \dot{\hat{a}}_1 = \beta_1 \Delta x(t) x^T(t - \tau_1),$$

$$\dot{\hat{b}}_0 = \gamma_0 \Delta x(t) u^T(t). \tag{32}$$

In the simulation, performed with MATLAB/SIMULINK 3.0, the following parameters

$$a_0 = -1.5,\ a_1 = 0.5,\ b_0 = 1,\ \tau_1 = 1 \tag{33}$$

of the system were selected. The initial distribution of the system and that of the identifier as well as the initial conditions of the tunable parameters were set to zero:

$$\begin{aligned} x(t) &= \hat{x}(t) = 0\ for\ all\ t \leq 0, \\ \hat{a}_i(0) &= 0,\ \hat{b}_i(0) = 0,\ i = 0, \ldots, \hat{r}. \end{aligned}$$

All of the adaptation parameters were set to eight.

We simulated two cases for identifier (31), (32), an ideal case with no uncertainties on the state delay, and a mismatched case with an approximate knowledge of the state delay. In these simulations two kinds of input signals, discontinuous square waves and smooth sine functions, were utilized. The simulation results are depicted in Figures 1-4.

Figures 1 and 2 compare the performance of the identifier in regard to whether sufficiently or insufficiently discontinuous and, respectively, rich input signals are applied to the identified system (28). From these figures the required convergence

$$J(t) = \frac{\sqrt{[\Delta a_0]^2(t) + [\Delta a_1]^2(t) + [\Delta b_0]^2(t)}}{3} \to 0\ as\ t \to \infty$$

of the parameter errors

$$\Delta a_0(t) = a_0 - \hat{a}_0(t),\ \Delta a_1(t) = a_1 - \hat{a}_1(t),\ \Delta b_0(t) = b_0 - \hat{b}_0(t)$$

is only concluded for sufficiently discontinuous and sufficiently rich input signals whereas the tunable parameters appear to converge to wrong values while applying insufficiently discontinuous and insufficiently rich input signals.

For the sake of comparison, different sufficiently discontinuous and sufficiently rich input signals were applied. From Figures 1 and 2 better identifier performance is concluded for the sufficiently discontinuous and sufficiently rich input signals with a greater number of incommensurable discontinuity points and that of frequencies, respectively.

In the numerical simulations, made for the parameter identifier (31), (32) in the mismatched case, the following model delays $\tau_1 = 1.01$ and $\tau_1 = 1.1$, both mismatching to that (33) of the system, were selected. The simulation results, shown in Figures 3 and 4, demonstrated strong robustness properties of the parameter identifier against potential uncertainties on the system delays.

These properties support the identification of the system delays by constructing the parameter identifier (29), (30). In our simulation we have simplified this identifier by setting

$$\hat{r} = 2,\ \hat{\tau}_1 = 1.01,\ \hat{\tau}_2 = 1.1,\ \hat{b}_1 = \hat{b}_2 = 0$$

and omitting (see Remark 2) the equations corresponding to the parameters $\hat{b}_1, \hat{b}_2$. As shown in Figures 5 and 6, the tunable parameters converge to their nominal values. However, compared to the implementation of the sufficiently discontinuous square wave input signal (Fig. 5) we have needed to apply one more frequency while implementing a combination of sine input signals (Fig. 6) to ensure its sufficient richness for the identification of the additional parameter a_2. To explain this one should take into account that, in contrast to a finite combination of sine signals, a nonsmooth (particularly, discontinuous) input signal, expanded into Fourier series, contains infinitely many frequencies.

It is concluded from Figs. 5 and 6 that the only parameter $\hat{a}_2(t)$ becomes neglectable as $t \to \infty$ thereby establishing a fictitious delay $\hat{\tau}_2 = 1.1$ and approximately identifying another delay value $\hat{\tau}_2 = 1.01$.

5 Conclusions

This paper presents on-line parameter identification of linear dynamic systems with finitely many delays in the states and inputs. It is assumed that the structure of the underlying system is known and only system parameters and delays are unknown. The state of the system is also assumed to be available for measurement.

Such a system is in principle identifiable if and only if it is weakly controllable. This constructively verifiable identifiability condition is combined with constructively enforceable sufficiently nonsmooth input signals into a practical framework for adaptive identification of the linear time-delay systems with unknown parameters and delays. The adaptive identifier of the system parameters and delays is represented as error systems, describing the evolution of the state error and parameter errors. The error systems take the form of functional differential equations with distributed delays. To guarantee the existence of a unique zero steady state for the parameter errors sufficiently nonsmooth periodic signals are constructed independently of any particular underlying system. If such a signal is applied to the system, the tunable parameters in the adaptive identifier are shown to converge to their nominal values. Theoretical results are supported by numerical simulation made for a scalar linear differential system with the only delay in the state and four unknown parameters, including the value of the state delay.

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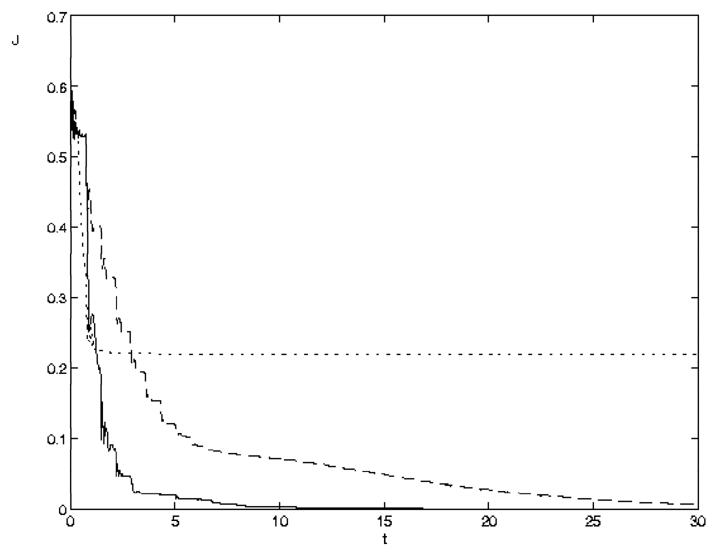

Fig. 1. Performance comparison for sufficiently and insufficiently discontinuous input signals: Plots of $J(t)$ for the square wave inputs with the frequencies a) $\omega_1 = 0.3$, $\omega_2 = 0.7$ and the corresponding amplitudes $M_1 = M_2 = 5$ (solid line), b) $\omega_1 = 0.3$, $\omega_2 = 0.7$ and the corresponding amplitudes $M_1 = 0$, $M_2 = 5$ (dashed line), c) $\omega_1 = 1$, $\omega_2 = 2$ and the corresponding amplitudes $M_1 = M_2 = 5$ (dotted line).

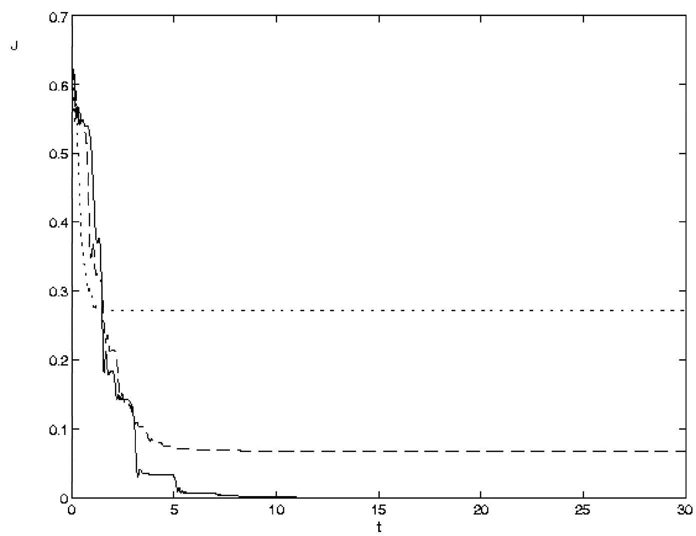

Fig. 2. Performance comparison for sufficiently and insufficiently rich input signals: Plots of $J(t)$ for the sine inputs with the frequencies a) $\omega_1 = 0.3$, $\omega_2 = 0.7$ and the corresponding amplitudes $M_1 = M_2 = 5$ (solid line), b) $\omega_1 = 0.3$, $\omega_2 = 0.7$ and the corresponding amplitudes $M_1 = 0$, $M_2 = 5$ (dashed line), c) $\omega_1 = 1$, $\omega_2 = 2$ and the corresponding amplitudes $M_1 = M_2 = 5$ (dotted line).

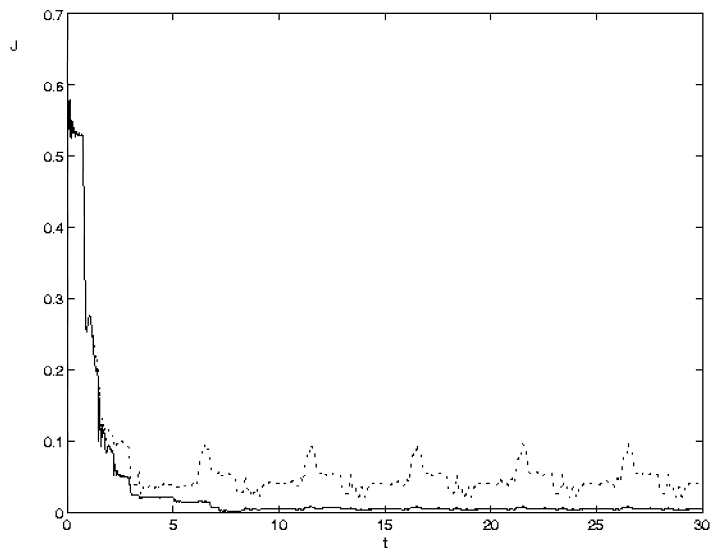

Fig. 3. Robustness against uncertainties on the delays: Plots of $J(t)$ for the square wave input with the frequencies $\omega_1 = 0.3$, $\omega_2 = 0.7$ and the corresponding amplitudes $M_1 = M_2 = 5$, and the model delay a) $\hat{\tau}_1 = 1.01$ (solid line), and b) $\hat{\tau}_1 = 1.1$ (dotted line).

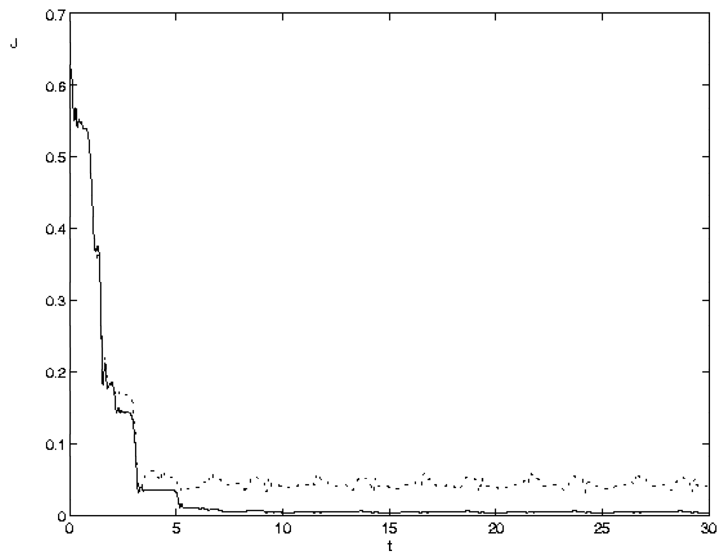

Fig. 4. Robustness against uncertainties on the delays: Plots of $J(t)$ for the sine input with the frequencies $\omega_1 = 0.3$, $\omega_2 = 0.7$ and the corresponding amplitudes $M_1 = M_2 = 5$, and the model delays $\tau_1 = 1.01$ (solid line) and $\tau_2 = 1.1$ (dotted line).

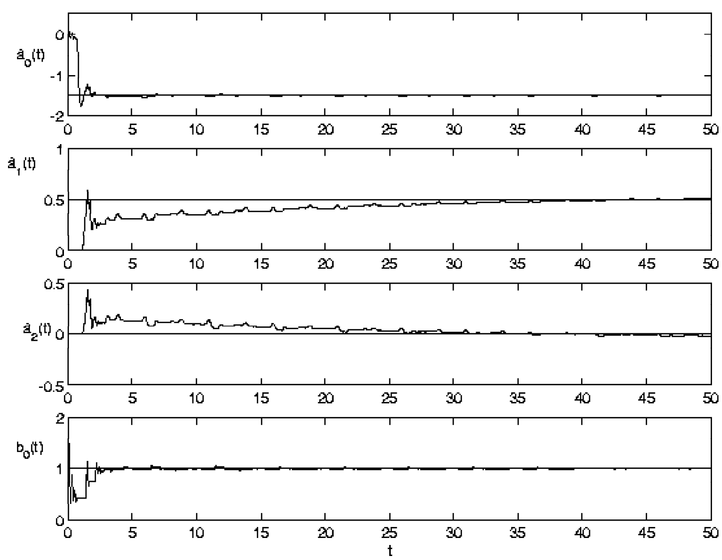

Fig. 5. Simultaneous parameter/delay identification: Plots of the tuned parameters $\hat{a}_0(t)$, $\hat{a}_1(t)$, $\hat{a}_2(t)$, $\hat{b}_0(t)$ for the square wave input with the frequencies $\omega_1 = 0.3$, $\omega_2 = 0.7$ and the corresponding amplitudes $M_1 = M_2 = 5$, and the model delays $\hat{\tau}_1 = 1.01$ and $\hat{\tau}_2 = 1.1$.

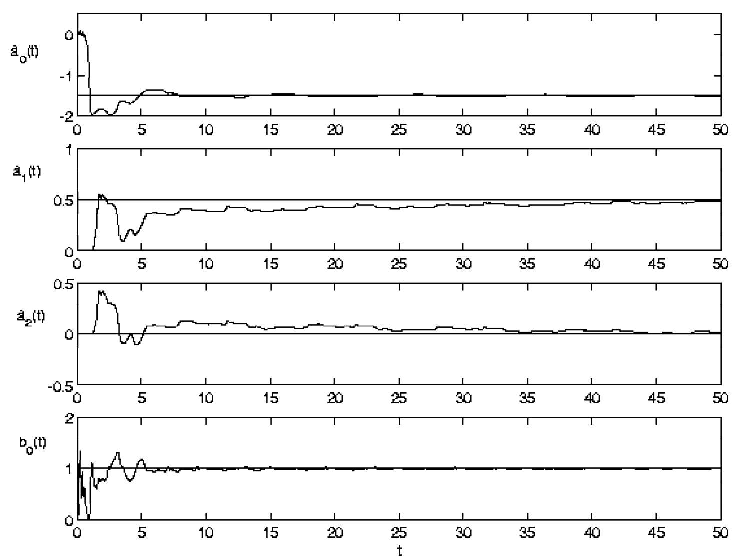

Fig. 6. Simultaneous parameter/delay identification: Plots of the tuned parameters $\hat{a}_0(t)$, $\hat{a}_1(t)$, $\hat{a}_2(t)$, $\hat{b}_0(t)$ for the sine input with the frequencies $\omega_1 = 0.3$, $\omega_2 = 0.7$ and the corresponding amplitudes $M_1 = M_2 = 5$, and the model delays $\hat{\tau}_1 = 1.01$ and $\hat{\tau}_2 = 1.1$.