New Control Schemes for Bilateral Teleoperation under Asymmetric Communication Channels: Stabilization and Performance under Variable Time Delays

Bo Zhang

LAGIS, CNRS UMR 8219, Laboratoire d'Automatique, Génie Informatique et Signal - Ecole Centrale de Lille - France
Thesis supervisor: Prof. Jean-Pierre Richard - EC Lille
Thesis co-advisor: Dr. Alexandre Kruszewski - EC Lille

10/07/2012

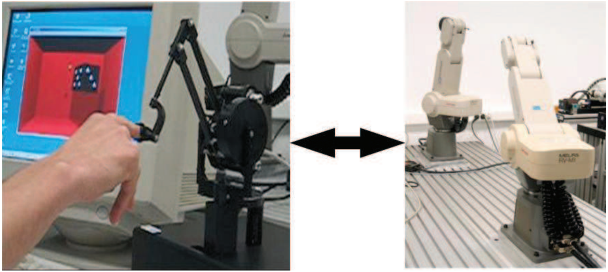
PLAN

1. PRELIMINARIES
2. NOVEL CONTROL SCHEMES
3. ROBUSTNESS ASPECTS
4. EXPERIMENTATIONS
5. CONCLUSIONS & PERSPECTIVES

1. Preliminaries

- ☺ Background & Challenges ;
- ☺ Delayed Teleoperation ;
- ☺ Positioning ;
- ☺ Modeling of Time Delays ;
- ☺ Stability of Time Delay Systems ;

Cooperative System :

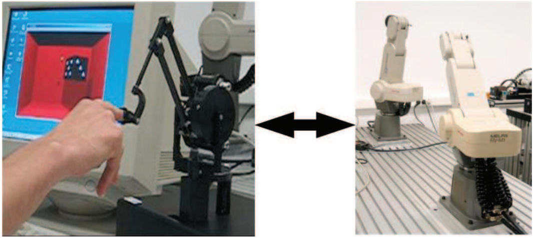


Haptic interface cooperative system, LAGIS

Performance Objectives :

☺ Stability ;
☺ Synchronization (the position tracking, from the master to the slave) ;
☺ Transparency (the force tracking, from the slave to the master) ;

Cooperative System :



Haptic interface cooperative system, LAGIS

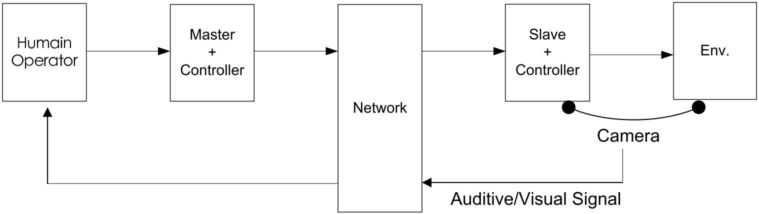
Background & Challenges :

- ☺ Wireless or long-distance ;
- ☺ Complex environment ;
- ☺ Modeling uncertainties ;
- ☺ Collaborative sensing ;
- ☺ Path planning and trajectory tracking ;

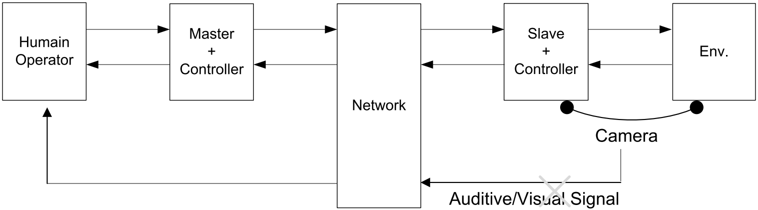
1. Preliminaries

- ☺ Background & Challenges ;
- ☺ Delayed Teleoperation ;
- ☺ Positioning ;
- ☺ Modeling of Time Delays ;
- ☺ Stability of Time Delay Systems ;

Delayed Teleoperation - Two Cases :

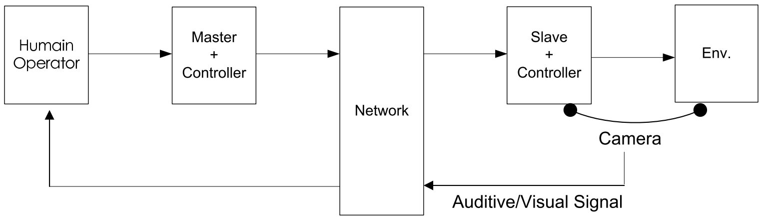


Unilateral master/slave teleoperation

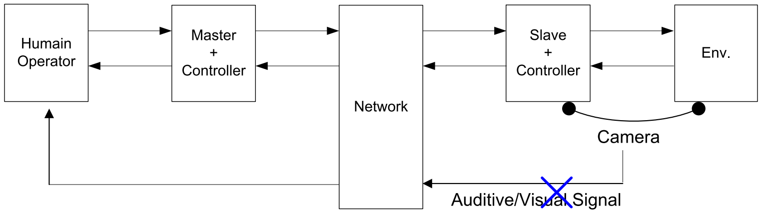


Bilateral master/slave teleoperation

Delayed Teleoperation - Two Cases :

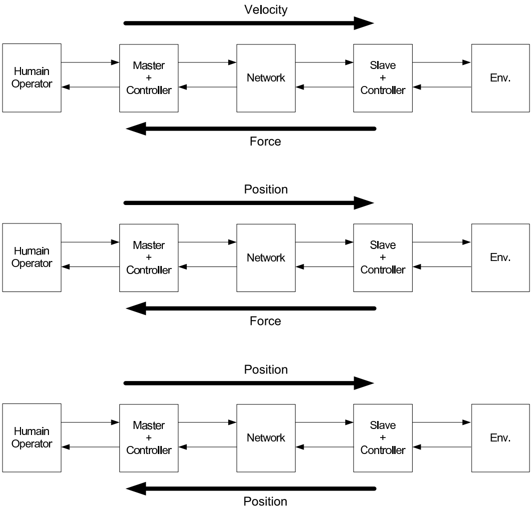


Unilateral master/slave teleoperation

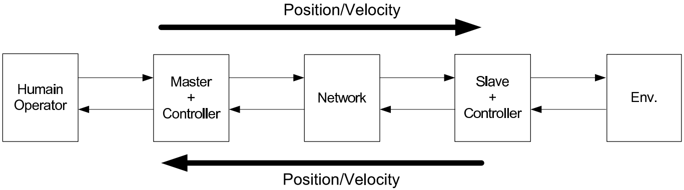


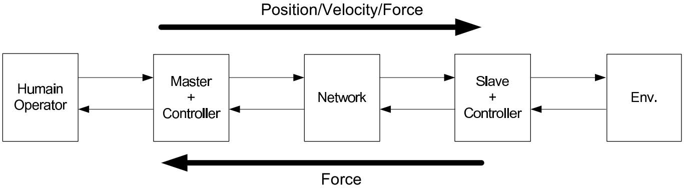
Bilateral master/slave teleoperation

Bilateral Teleoperation :

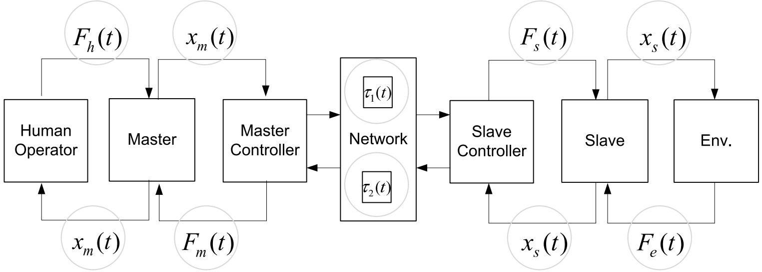


Bilateral Teleoperation :



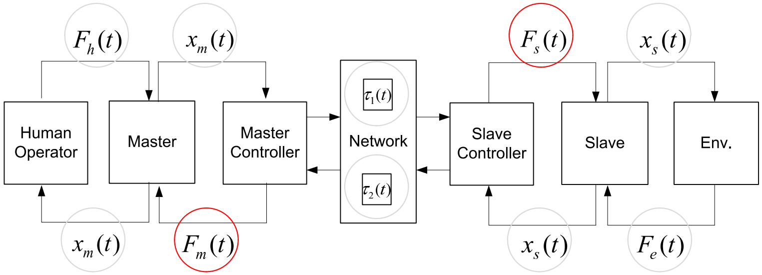


General Teleoperation Structure :



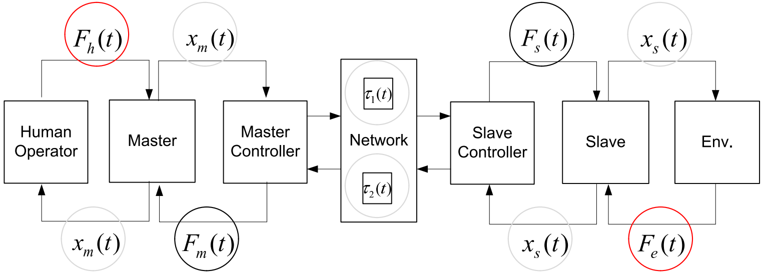
- ☺ $F \Longrightarrow$ force ;
- ☺ $x \Longrightarrow$ position/velocity ;
- ☺ $\tau \Longrightarrow$ delay ;
- ☺ $m,\ s,\ h,\ e \Longrightarrow$ master, slave, human operator, environment ;

General Teleoperation Structure :



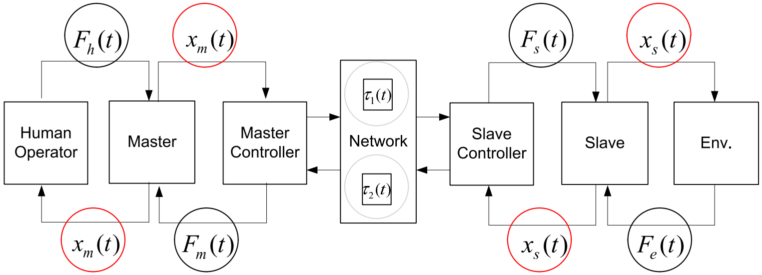
- ☺ $F \Longrightarrow$ force ;
- ☺ $x \Longrightarrow$ position/velocity ;
- ☺ $\tau \Longrightarrow$ delay ;
- ☺ $m,\ s,\ h,\ e \Longrightarrow$ master, slave, human operator, environment ;

General Teleoperation Structure :



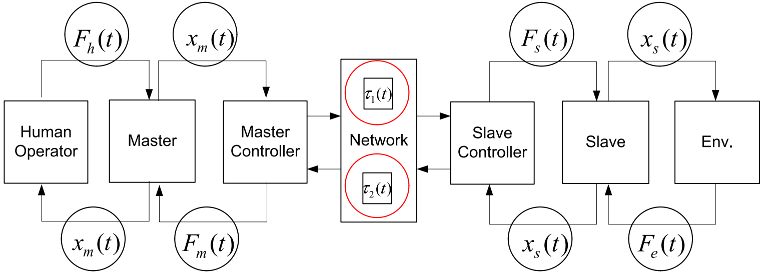
- ☺ $F \Longrightarrow$ force ;
- ☺ $x \Longrightarrow$ position/velocity ;
- ☺ $\tau \Longrightarrow$ delay ;
- ☺ $m,\ s,\ h,\ e \Longrightarrow$ master, slave, human operator, environment ;

General Teleoperation Structure :



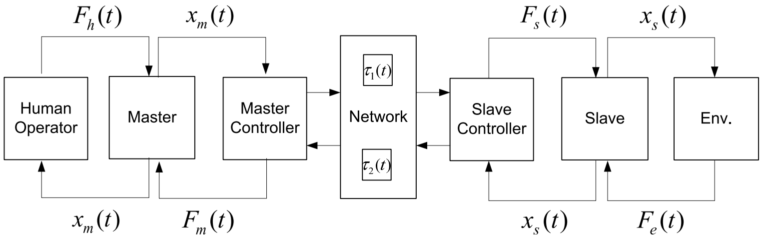
- ☺ $F \Longrightarrow$ force ;
- ☺ $x \Longrightarrow$ position/velocity ;
- ☺ $\tau \Longrightarrow$ delay ;
- ☺ $m,\ s,\ h,\ e \Longrightarrow$ master, slave, human operator, environment ;

General Teleoperation Structure :



- ☺ $F \Longrightarrow$ force ;
- ☺ $x \Longrightarrow$ position/velocity ;
- ☺ $\tau \Longrightarrow$ delay ;
- ☺ $m, s, h, e \Longrightarrow$ master, slave, human operator, environment ;

General Teleoperation Structure :



Performance Objectives :

☺ Stability ;
☺ Synchronization (the position tracking, from the master to the slave) ;
☺ Transparency (the force tracking, from the slave to the master) ;

Properties :

1. Linear master/slave systems ;
2. Internet/Ethernet/Wifi... ;
3. Time-stamped data packets ;

1. Preliminaries

- ☺ Background & Challenges ;
- ☺ Delayed Teleoperation ;
- ☺ Positioning ;
- ☺ Modeling of Time Delays ;
- ☺ Stability of Time Delay Systems ;

Main Capabilities of Recent Control Strategies :

Control Strategy	Time Delays		Position Tracking	Force Tracking
	Constant	Time-varying		
Passivity	√	√	√	
Robust	√	√	√	√
Freq.	√		√	
Predict.	√	√	√	
SMC	√	√	√	
Adapt.	√		√	
Lyapunov	√	√	√	

- ☹ *P. Arcara et al., Robotics and Autonomous Systems, 2002* ;
- ☹ *S. Zampieri, IFAC, 2008* ;
- ☹ *E. Nuño et al., Automatica, 2011* ;

Control Strategy	Time Delays		Position Tracking	Force Tracking	
	Constant	Time-varying			
Goals of this thesis	√	√	√	√	+ Performance

Main Capabilities of Recent Control Strategies :

Control Strategy	Time Delays		Position Tracking	Force Tracking
	Constant	Time-varying		
Passivity	√	√	√	
Robust	√	√	√	√
Freq.	√		√	
Predict.	√	√	√	
SMC	√	√	√	
Adapt.	√		√	
Lyapunov	√	√	√	

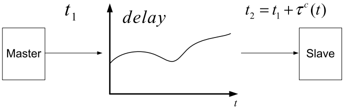
- ☺ *P. Arcara et al., Robotics and Autonomous Systems, 2002* ;
- ☺ *S. Zampieri, IFAC, 2008* ;
- ☺ *E. Nuño et al., Automatica, 2011* ;

Control Strategy	Time Delays		Position Tracking	Force Tracking	
	Constant	Time-varying			
Goals of this thesis	√	√	√	√	+ Performance

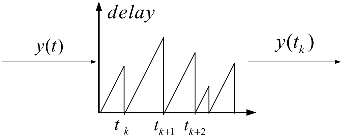
1. Preliminaries

- ☺ Background & Challenges ;
- ☺ Delayed Teleoperation ;
- ☺ Positioning ;
- ☺ Modeling of Time Delays ;
- ☺ Stability of Time Delay Systems ;

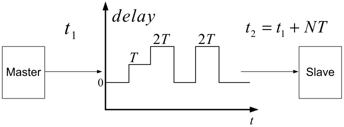
Network Induced Delays :



Communication delay $\tau^c(t)$

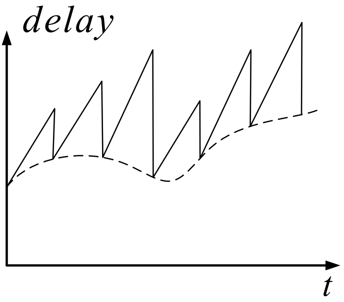


Asynchronous sampling delay $\tau^s(t)$



Data loss delay

Network Induced Delays :



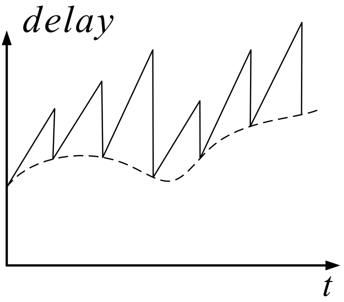
$$\tau(t) = \tau^c(t) + \tau^s(t) + NT. \quad (1)$$

Notations and Constraints for Delays :

$$\tau(t) = h + \eta(t). \quad (2)$$

- Constant delay : $\eta(t) = 0,\ h \neq 0$;
- Non-small time-varying delay : $\mid \eta(t) \mid \leqslant \mu < h$, so $\tau(t) \in [h - \mu, h + \mu]$;
- Interval time-varying delay : $0 \leqslant \eta(t) \leqslant \mu$, so $\tau(t) \in [h, h + \mu]$;
- Time delay with the constraint on the derivative : $\dot{\tau}(t) \leqslant d < 1$, $d > 0$, or $\dot{\tau}(t) \leqslant 1$;

Network Induced Delays :



$$\tau(t) = \tau^c(t) + \tau^s(t) + NT. \quad (1)$$

Notations and Constraints for Delays :

$$\tau(t) = h + \eta(t). \quad (2)$$

- Constant delay : $\eta(t) = 0,\ h \neq 0$;
- Non-small time-varying delay : $|\ \eta(t)\ | \leqslant \mu < h$, so $\tau(t) \in [h - \mu, h + \mu]$;
- Interval time-varying delay : $0 \leqslant \eta(t) \leqslant \mu$, so $\tau(t) \in [h, h + \mu]$;
- Time delay with the constraint on the derivative : $\dot{\tau}(t) \leqslant d < 1$, $d > 0$, or $\dot{\tau}(t) \leqslant 1$;

1. Preliminaries

- ☺ Background & Challenges ;
- ☺ Delayed Teleoperation ;
- ☺ Positioning ;
- ☺ Modeling of Time Delays ;
- ☺ Stability of Time Delay Systems ;

Delay-Independent Stability :

If a time-delay system is asymptotically stable for any delay values belonging to $\mathbb{R}_+$, the system is said to be delay-independent asymptotically stable.

Delay-Dependent Stability :

If a time-delay system is asymptotically stable for all delay values belonging to a compact subset D of $\mathbb{R}_+$, the system is said to be delay-dependent asymptotically stable.

Rate-Independent Stability :

For a delay-dependent asymptotically stable time delay system, if the stability does not depend on the variation rate of delays or on the time derivative of delays, the system is said to be rate-independent asymptotically stable.

Rate-Dependent Stability :

For a delay-dependent asymptotically stable time delay system, if the stability depends on the variation rate of delays or on the time derivative of delays, the system is said to be rate-dependent asymptotically stable.

Delay-Independent Stability :

If a time-delay system is asymptotically stable for any delay values belonging to $\mathbb{R}_+$, the system is said to be delay-independent asymptotically stable.

Delay-Dependent Stability :

If a time-delay system is asymptotically stable for all delay values belonging to a compact subset D of $\mathbb{R}_+$, the system is said to be delay-dependent asymptotically stable.

Rate-Independent Stability :

For a delay-dependent asymptotically stable time delay system, if the stability does not depend on the variation rate of delays or on the time derivative of delays, the system is said to be rate-independent asymptotically stable.

Rate-Dependent Stability :

For a delay-dependent asymptotically stable time delay system, if the stability depends on the variation rate of delays or on the time derivative of delays, the system is said to be rate-dependent asymptotically stable.

General System with Piecewise Time-Varying Delays :

$$\begin{cases} \dot{x}(t) = f(x(t), x(t-\tau_1(t)), u(t), u(t-\tau_2(t))), \\ x(t_0+\theta) = \phi(\theta), \quad \dot{x}(t_0+\theta) = \dot{\phi}(\theta), \\ u(t_0+\theta) = \zeta(\theta), \\ \theta \in [-h, 0]. \end{cases} \tag{3}$$

Linear Case :

$$\begin{cases} \dot{x}(t) = A_0 x(t) + \sum_{i=1}^{n} A_i x(t-\tau_{1i}(t)) + B_0 u(t) + \sum_{j=1}^{m} B_j u(t-\tau_{2j}(t)), \\ x(t_0+\theta) = \phi(\theta), \quad \dot{x}(t_0+\theta) = \dot{\phi}(\theta), \quad \theta \in [-h, 0]. \end{cases} \tag{4}$$

Lyapunov-Krasovskii Functionals (LKF) - Linear Case :

☺ Search of suitable $V(x(t), \dot{x}(t))$ *[E. Fridman, IMA Journal of Mathematical Control and Information, 2006]* :

$$V(x(t), \dot{x}(t)) = x(t)^T P x(t) + \int_{t-h_2}^{t} x(s)^T S_a x(s) ds + \int_{t-h_1}^{t} x(s)^T S x(s) ds$$
$$+ h_1 \int_{-h_1}^{0} \int_{t+\theta}^{t} \dot{x}(s)^T R \dot{x}(s) ds d\theta$$
$$+ \sum_{i=1}^{q} (h_2 - h_1) \int_{-h_2}^{-h_1} \int_{t+\theta}^{t} \dot{x}(s)^T R_{ai} \dot{x}(s) ds d\theta. \quad (5)$$

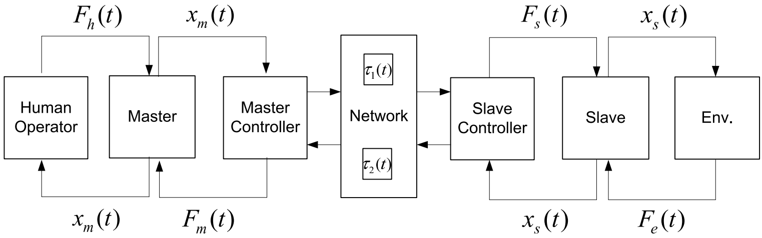
☺ Asymptotical stability condition depending of the derivative of $V(x(t), \dot{x}(t))$ along the system trajectories :

$$\begin{aligned} &V(x(t), \dot{x}(t)) > 0, \quad \dot{V}(x(t), \dot{x}(t)) < 0, \quad for \quad any \quad x_t \neq 0, \\ &and \quad V(x(0), \dot{x}(0)) = 0, \quad \dot{V}(x(0), \dot{x}(0)) = 0. \end{aligned} \quad (6)$$

2. Novel Control Schemes

- ☺ System Description & Assumptions ;
- ☺ Robust Stability Conditions ;
- ☺ Control Scheme 1 - Bilateral State Feedback Control Scheme *[B. Zhang et al., CCDC, 2011]* ;
- ☺ Control Scheme 2 - Force-Reflecting Control Scheme ;
- ☺ Control Scheme 3 - Force-Reflecting Proxy Control Scheme *[B. Zhang et al., ETFA, 2011]* ;
- ☺ Results and Analysis ;

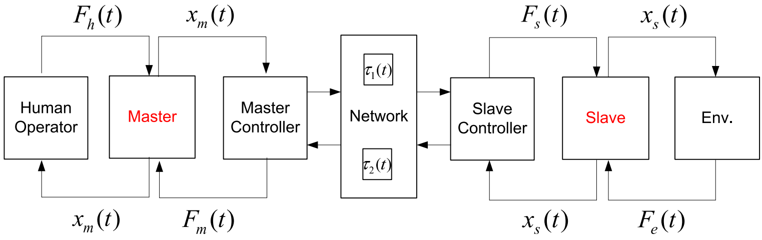
General Teleoperation Structure :



Our Master/Slave Controller Solution :

- ☺ Lyapunov-Krasovskii functional (LKF) ;
- ☺ H_∞ control ;
- ☺ Linear Matrix Inequality (LMI) optimization ;

General Teleoperation Structure :



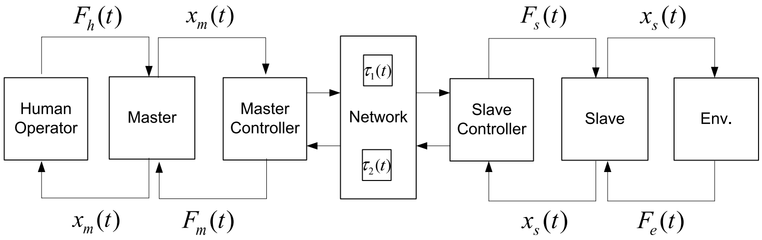
From Property 1 - Assumption 1 :

Linear master/slave systems $\Longrightarrow$

$$\begin{aligned} \dot{x}_m(t) &= (A_m - B_m K_0^m)x_m(t) + B_m(F_m(t) + F_h(t)), \\ \dot{x}_s(t) &= (A_s - B_s K_0^s)x_s(t) + B_s(F_s(t) + F_e(t)), \end{aligned} \tag{7}$$

$x_m(t) = \dot{\theta}_m(t) \in \mathbb{R}^n$, $x_s(t) = \dot{\theta}_s(t) \in \mathbb{R}^n$; K_0^m & K_0^s : supposed to be known ;

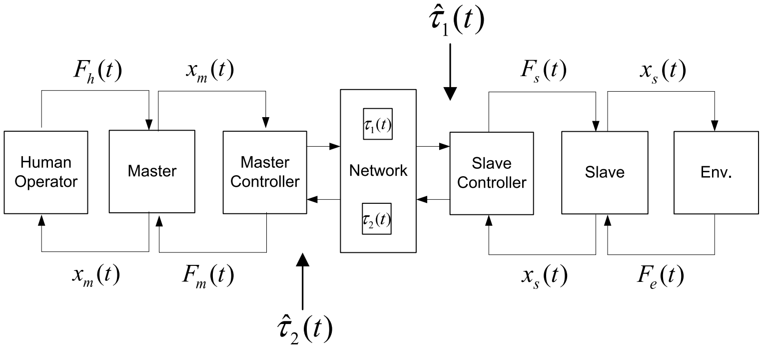
General Teleoperation Structure :



From Property 2 - Assumption 2 :

Internet/Ethernet/Wifi... $\Longrightarrow \tau_1(t),\ \tau_2(t) \in [h_1, h_2],\ h_1 \geq 0\,;$

General Teleoperation Structure :



From Property 3 - Assumption 3 :

Time-stamped data packets $\Longrightarrow \hat{\tau}_1(t) = \tau_1(t),\ \hat{\tau}_2(t) = \tau_2(t)$;

2. Novel Control Schemes

- ☺ System Description & Assumptions ;
- ☺ Robust Stability Conditions ;
- ☺ Control Scheme 1 - Bilateral State Feedback Control Scheme ;
  [B. Zhang et al., CCDC, 2011] ;
- ☺ Control Scheme 2 - Force-Reflecting Control Scheme ;
- ☺ Control Scheme 3 - Force-Reflecting Proxy Control Scheme
  [B. Zhang et al., ETFA, 2011] ;
- ☺ Results and Analysis ;

Linear Time Delay System :

$$\begin{cases} \dot{x}(t) = A_0 x(t) + \sum_{i=1}^{q} A_i x(t - \tau_i(t)), \\ x(t_0 + \theta) = \phi(\theta), \quad \dot{x}(t_0 + \theta) = \dot{\phi}(\theta), \quad \theta \in [-h_2, 0]. \end{cases} \tag{8}$$

Asymptotical Stability Theorem *[E. Fridman, IMA Journal of Mathematical Control and Information, 2006]* :

☺ $P > 0$, $R > 0$, $S > 0$, $S_a > 0$, $R_{ai} > 0$, and P_2, P_3, Y_1, Y_2, $i = 1, 2, ..., q$;
☺ LMI condition is feasible;
☺ Rate-independent asymptotically stable for time-varying delays $\tau_i(t) \in [h_1, h_2]$, $i = 1, 2, ..., q$;

Asymptotical Stability Theorem :

$$\Gamma^1 = \begin{pmatrix} \Gamma^1_{11} & \Gamma^1_{12} & R+\sum_{i=1}^{q} P_2^T A_i - qY_1^T & qY_1^T & -P_2^T A_1 + Y_1^T & \dots & -P_2^T A_q + Y_1^T & Y_1^T & \dots & Y_1^T \\ \ast & \Gamma^1_{22} & \sum_{i=1}^{q} P_3^T A_i - qY_2^T & qY_2^T & -P_3^T A_1 + Y_2^T & \dots & -P_3^T A_q + Y_2^T & Y_2^T & \dots & Y_2^T \\ \ast & \ast & -S-R & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \ast & \ast & \ast & -S_a & 0 & 0 & 0 & 0 & 0 & 0 \\ \ast & \ast & \ast & \ast & -R_{a1} & 0 & 0 & 0 & 0 & 0 \\ \ast & \ast & \ast & \ast & \ast & \dots & 0 & 0 & 0 & 0 \\ \ast & \ast & \ast & \ast & \ast & \ast & -R_{aq} & 0 & 0 & 0 \\ \ast & \ast & \ast & \ast & \ast & \ast & \ast & -R_{a1} & 0 & 0 \\ \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \dots & 0 \\ \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -R_{aq} \end{pmatrix} < 0, \quad (9)$$

$$\Gamma^1_{11} = S + S_a - R + A_0^T P_2 + P_2^T A_0, \quad \Gamma^1_{12} = P - P_2^T + A_0^T P_3,$$
$$\Gamma^1_{22} = -P_3 - P_3^T + h_1^2 R + (h_2 - h_1)^2 \sum_{i=1}^{q} R_{ai}. \quad (10)$$

H_∞ Performance - Delay-Free with Perturbation :

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bw(t), \\ z(t) &= Cx(t). \end{cases} \tag{11}$$

$$J(w) = \int_0^\infty (z(t)^T z(t) - \gamma^2 w(t)^T w(t))dt < 0, \\ (sup_w(\frac{\| z(t) \|_2}{\| w(t) \|_2}) < \gamma). \tag{12}$$

$$\dot{V}(x(t)) + z(t)^T z(t) - \gamma^2 w(t)^T w(t) < 0, \; V(x(t)) = x(t)^T P x(t).$$

Robust Stability Theorem :

- ☺ $P > 0$, P_2, P_3, and a positive scalar $\gamma > 0$;
- ☺ LMI condition is feasible (see manuscript) ;
- ☺ Asymptotically stable with H_∞ performance $J(w) < 0$;

H_∞ Performance - Delay-Free with Perturbation :

$$\begin{cases} \dot{x}(t) & = Ax(t) + Bw(t), \\ z(t) & = Cx(t). \end{cases} \tag{11}$$

$$J(w) = \int_0^\infty (z(t)^T z(t) - \gamma^2 w(t)^T w(t))dt < 0, \quad (sup_w(\frac{\| z(t) \|_2}{\| w(t) \|_2}) < \gamma). \tag{12}$$

$\dot{V}(x(t)) + z(t)^T z(t) - \gamma^2 w(t)^T w(t) < 0$, $V(x(t)) = x(t)^T P x(t)$.

Robust Stability Theorem :

- ☺ $P > 0$, P_2, P_3, and a positive scalar $\gamma > 0$;
- ☺ LMI condition is feasible (see manuscript) ;
- ☺ Asymptotically stable with H_∞ performance $J(w) < 0$;

H_∞ Performance - Delay-Free with Perturbation :

$$\begin{cases} \dot{x}(t) & = Ax(t) + Bw(t), \\ z(t) & = Cx(t). \end{cases} \tag{11}$$

$$J(w) = \int_0^\infty (z(t)^T z(t) - \gamma^2 w(t)^T w(t))dt < 0, \quad (sup_w(\frac{\| z(t) \|_2}{\| w(t) \|_2}) < \gamma). \tag{12}$$

$\dot{V}(x(t)) + z(t)^T z(t) - \gamma^2 w(t)^T w(t) < 0$, $V(x(t)) = x(t)^T P x(t)$.

Robust Stability Theorem :

- ☺ $P > 0$, P_2, P_3, and a positive scalar $\gamma > 0$;
- ☺ LMI condition is feasible (see manuscript) ;
- ☺ Asymptotically stable with H_∞ performance $J(w) < 0$;

Robust Stability Condition - Teleoperation Case :

☺ LKF asymptotical stability condition with several time-varying delays

+

☺ H_∞ performance improvement condition without time-varying delays but with the perturbation

=

☺ Robust stability condition with several time-varying delays and the perturbation :

$$\begin{cases} \dot{x}(t) = A_0 x(t) + \sum_{i=1}^{q} A_i x(t - \tau_i(t)) + Bw(t), \\ z(t) = Cx(t), \\ x(t_0 + \theta) = \phi(\theta), \quad \dot{x}(t_0 + \theta) = \dot{\phi}(\theta), \quad \theta \in [-h_2, 0], \end{cases} \tag{13}$$

$$\dot{V}(x(t), \dot{x}(t)) + z(t)^T z(t) - \gamma^2 w(t)^T w(t) < 0.$$

Robust Stability Condition - Teleoperation Case :

☺ LKF asymptotical stability condition with several time-varying delays

$+$

☺ H_∞ performance improvement condition without time-varying delays but with the perturbation

$=$

☺ Robust stability condition with several time-varying delays and the perturbation :

$$\begin{cases} \dot{x}(t) = A_0 x(t) + \sum_{i=1}^{q} A_i x(t-\tau_i(t)) + Bw(t), \\ z(t) = Cx(t), \\ x(t_0+\theta) = \phi(\theta), \quad \dot{x}(t_0+\theta) = \dot{\phi}(\theta), \quad \theta \in [-h_2, 0], \end{cases} \tag{13}$$

$$\dot{V}(x(t), \dot{x}(t)) + z(t)^T z(t) - \gamma^2 w(t)^T w(t) < 0.$$

Robust Stability Condition - Teleoperation Case :

☺ LKF asymptotical stability condition with several time-varying delays

+

☺ H_∞ performance improvement condition without time-varying delays but with the perturbation

=

☺ Robust stability condition with several time-varying delays and the perturbation :

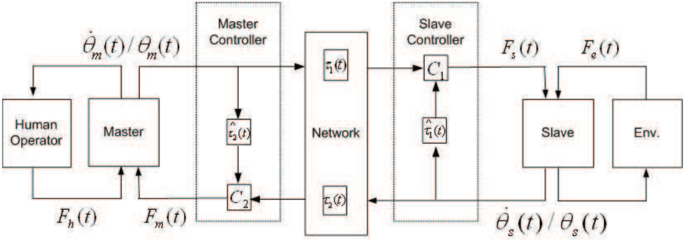
$$\begin{cases} \dot{x}(t) = A_0 x(t) + \sum_{i=1}^{q} A_i x(t - \tau_i(t)) + Bw(t), \\ z(t) = Cx(t), \\ x(t_0 + \theta) = \phi(\theta), \quad \dot{x}(t_0 + \theta) = \dot{\phi}(\theta), \quad \theta \in [-h_2, 0], \end{cases} \tag{13}$$

$$\dot{V}(x(t), \dot{x}(t)) + z(t)^T z(t) - \gamma^2 w(t)^T w(t) < 0.$$

2. Novel Control Schemes

- ☺ System Description & Assumptions ;
- ☺ Robust Stability Conditions ;
- ☺ Control Scheme 1 - Bilateral State Feedback Control Scheme ;
  [B. Zhang et al., CCDC, 2011] ;
- ☺ Control Scheme 2 - Force-Reflecting Control Scheme ;
- ☺ Control Scheme 3 - Force-Reflecting Proxy Control Scheme
  [B. Zhang et al., ETFA, 2011] ;
- ☺ Results and Analysis ;

Control Scheme 1 - Bilateral State Feedback Control Scheme :



☺ Delayed state feedback C_1 & C_2 :

$$\begin{aligned} C_1: \quad F_s(t) &= -K_1^1\dot{\theta}_s(t-\hat{\tau}_1(t)) - K_1^2\dot{\theta}_m(t-\tau_1(t)) \\ &\quad - K_1^3(\theta_s(t-\hat{\tau}_1(t) - \theta_m(t-\tau_1(t))), \\ C_2: \quad F_m(t) &= -K_2^1\dot{\theta}_s(t-\tau_2(t)) - K_2^2\dot{\theta}_m(t-\hat{\tau}_2(t)) \\ &\quad - K_2^3(\theta_s(t-\tau_2(t) - \theta_m(t-\hat{\tau}_2(t))). \end{aligned} \tag{14}$$

☺ $\hat{\tau}_1(t) = \tau_1(t)$ and $\hat{\tau}_2(t) = \tau_2(t)$ from *Assumption 3*;

Control Scheme 1 - Master/Slave Controller Design :

$$\begin{cases} \dot{x}_{ms}(t) &= (A_{ms} - B_{ms}K_0)x_{ms}(t) + B_{ms}u_{ms}(t) + B_{ms}w_{ms}(t), \\ u_{ms}(t) &= -K^1_{ms}x_{ms}(t-\tau_1(t)) - K^2_{ms}x_{ms}(t-\tau_2(t)), \\ z_{ms}(t) &= C_{ms}x_{ms}(t), \end{cases} \tag{15}$$

$$x_{ms}(t) = \begin{pmatrix} \dot{\theta}_s(t) \\ \dot{\theta}_m(t) \\ \theta_s(t)-\theta_m(t) \end{pmatrix}, \quad u_{ms}(t) = \begin{pmatrix} F_s(t) \\ F_m(t) \end{pmatrix}, \quad w_{ms}(t) = \begin{pmatrix} F_e(t) \\ F_h(t) \end{pmatrix}, \quad z_{ms}(t) = \begin{pmatrix} \theta_s(t)-\theta_m(t) \end{pmatrix}, \tag{16}$$

$$A_{ms} = \begin{pmatrix} A_s & 0 & 0 \\ 0 & A_m & 0 \\ 1 & -1 & 0 \end{pmatrix}, \quad B_{ms} = \begin{pmatrix} B_s & 0 \\ 0 & B_m \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} B^1_{ms} & B^2_{ms} \end{pmatrix},$$
$$K_0 = \begin{pmatrix} K^s_0 & 0 & 0 \\ 0 & K^m_0 & 0 \end{pmatrix}, \quad K^1_{ms} = \begin{pmatrix} K^1_1 & K^2_1 & K^3_1 \\ 0 & 0 & 0 \end{pmatrix}, \quad K^2_{ms} = \begin{pmatrix} 0 & 0 & 0 \\ K^1_2 & K^2_2 & K^3_2 \end{pmatrix}, \tag{17}$$
$$C_{ms} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}.$$

Control Scheme 1 - Master/Slave Controller Design :

☺ Transformation so to apply robust control condition :

$$\left\{\begin{array}{ll} \dot{x}_{ms}(t) & = A^0_{ms}x_{ms}(t) + A^1_{ms}x_{ms}(t-\tau_1(t)) + A^2_{ms}x_{ms}(t-\tau_2(t)) \\ & +B_{ms}w_{ms}(t), \\ z_{ms}(t) & = C_{ms}x_{ms}(t), \end{array}\right. \tag{18}$$

$$\begin{array}{l} A^0_{ms} = A_{ms} - B_{ms}K_0, \\ A^1_{ms} = -B_{ms}K^1_{ms} = -B^1_{ms}K_1, \quad A^2_{ms} = -B_{ms}K^2_{ms} = -B^2_{ms}K_2, \end{array} \tag{19}$$

$$K_1 = \begin{pmatrix} K^1_1 & K^2_1 & K^3_1 \end{pmatrix}, \quad K_2 = \begin{pmatrix} K^1_2 & K^2_2 & K^3_2 \end{pmatrix}. \tag{20}$$

Control Scheme 1 - Control Objectives :

☺ LKF : the system stability under the time-varying delays $\tau_1(t)$, $\tau_2(t) \in [h_1, h_2]$;
☺ H_∞ control : the impact γ of disturbances $w_{ms}(t)$ on $z_{ms}(t)$ $(\theta_s(t) - \theta_m(t))$;

Control Scheme 1 - Master/Slave Controller Design Theorem :

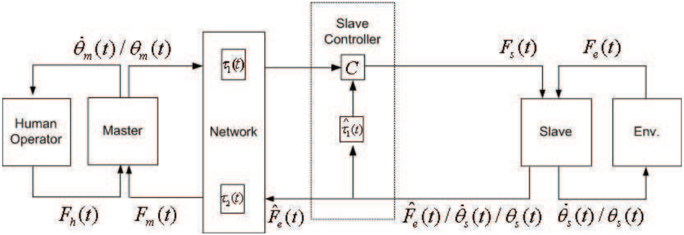
☺ $P > 0$, $R > 0$, $S > 0$, $S_a > 0$, $R_{a1} > 0$, $R_{a2} > 0$, P_2, W_1, W_2, Y_1, Y_2, and positive scalars γ and ξ ;
☺ LMI condition is feasible (see manuscript) ;
☺ Rate-independent asymptotically stable with H_∞ performance $J(w) < 0$ for time-varying delays $\tau_1(t)$, $\tau_2(t) \in [h_1, h_2]$:

$$K_1 = W_1 P_2^{-1}, \quad K_2 = W_2 P_2^{-1}. \tag{21}$$

2. Novel Control Schemes

- ☺ System Description & Assumptions ;
- ☺ Robust Stability Conditions ;
- ☺ Control Scheme 1 - Bilateral State Feedback Control Scheme *[B. Zhang et al., CCDC, 2011]* ;
- ☺ Control Scheme 2 - Force-Reflecting Control Scheme ;
- ☺ Control Scheme 3 - Force-Reflecting Proxy Control Scheme *[B. Zhang et al., ETFA, 2011]* ;
- ☺ Results and Analysis ;

Control Scheme 2 - Force-Reflecting Control Scheme :



- ☺ $\hat{F}_e(t)$: $F_m(t) = \hat{F}_e(t - \tau_2(t))$;
- ☺ C with controller gain K^i, $i = 1, 2, 3$:

$$C: \quad F_s(t) = -K^1\dot{\theta}_s(t - \hat{\tau}_1(t)) - K^2\dot{\theta}_m(t - \tau_1(t)) - K^3(\theta_s(t - \hat{\tau}_1(t) - \theta_m(t - \tau_1(t))). \quad (22)$$

Control Scheme 2 - Slave Controller Design :

$$\begin{cases} \dot{x}_{ms}(t) &= (A_{ms} - B_{ms}K_0)x_{ms}(t) + B_{ms}u_{ms}(t) + B_{ms}w_{ms}(t), \\ u_{ms}(t) &= -K_{ms}x_{ms}(t - \tau_1(t)), \end{cases} \tag{23}$$

$$K_{ms} = \begin{pmatrix} K^1 & K^2 & K^3 \\ 0 & 0 & 0 \end{pmatrix}. \tag{24}$$

$$\Downarrow$$

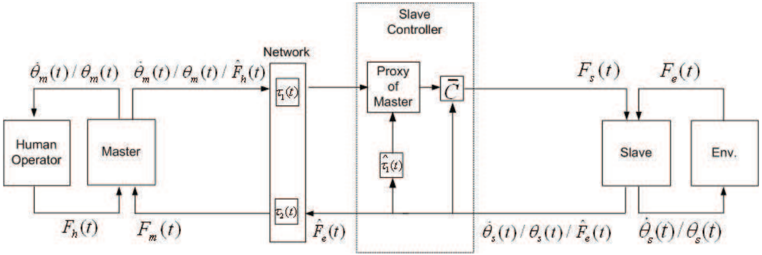
$$\begin{cases} \dot{x}_{ms}(t) &= A^0_{ms}x_{ms}(t) + A^1_{ms}x_{ms}(t - \tau_1(t)) + B_{ms}w_{ms}(t), \\ z_{ms}(t) &= C_{ms}x_{ms}(t), \end{cases} \tag{25}$$

$$A^1_{ms} = -B_{ms}K_{ms} = -B^1_{ms}K, \quad K = \begin{pmatrix} K^1 & K^2 & K^3 \end{pmatrix}. \tag{26}$$

2. Novel Control Schemes

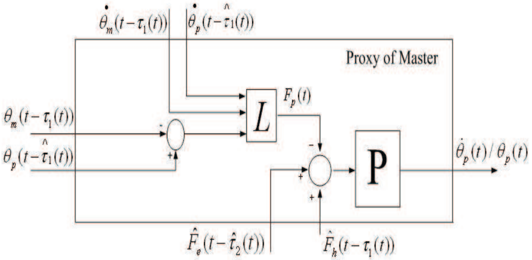
- ☺ System Description & Assumptions ;
- ☺ Robust Stability Conditions ;
- ☺ Control Scheme 1 - Bilateral State Feedback Control Scheme ;
  [B. Zhang et al., CCDC, 2011] ;
- ☺ Control Scheme 2 - Force-Reflecting Control Scheme ;
- ☺ Control Scheme 3 - Force-Reflecting Proxy Control Scheme
  [B. Zhang et al., ETFA, 2011] ;
- ☺ Results and Analysis ;

Control Scheme 3 - Force-Reflecting Proxy Control Scheme :



☺ From master to slave : $\dot{\theta}_m(t)/\theta_m(t)/\hat{F}_h(t)$, the position tracking ;
☺ From slave to master : $F_m(t) = \hat{F}_e(t - \tau_2(t))$, the force tracking ;

Control Scheme 3 - Slave Controller Description :

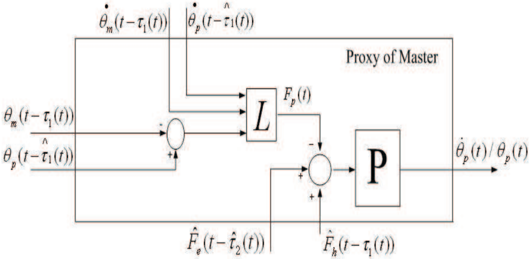


☺ 'P' :

$$\dot{x}_p(t) = (A_m - B_m K_0^m)x_p(t) - B_m F_p(t) + B_m(\hat{F}_e(t - \hat{\tau}_1(t)) + \hat{F}_h(t - \tau_1(t))), \tag{27}$$

$x_p(t) = \dot{\theta}_p(t) \in \mathbb{R}^n$.

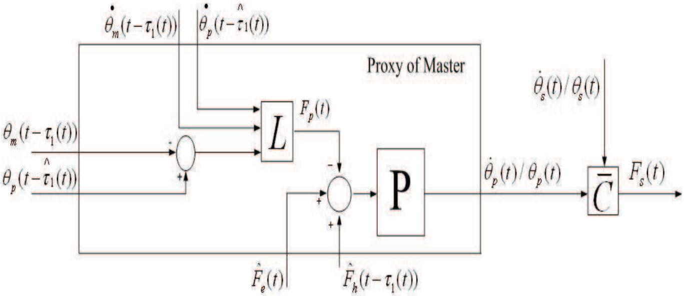
Control Scheme 3 - Slave Controller Description :



☺ $L = \begin{pmatrix} L_1 & L_2 & L_3 \end{pmatrix} \Longrightarrow \theta_p(t) \to \theta_m(t)$:

$$F_p(t) = L \begin{pmatrix} \dot{\theta}_p(t-\hat{\tau}_1(t)) \\ \dot{\theta}_m(t-\tau_1(t)) \\ \theta_p(t-\hat{\tau}_1(t))-\theta_m(t-\tau_1(t)) \end{pmatrix}. \quad (27)$$

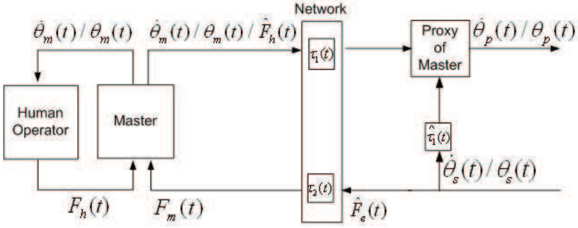
Control Scheme 3 - Slave Controller Description :



☺ $K = \begin{pmatrix} K_1 & K_2 & K_3 \end{pmatrix}$ of the controller $\bar{C} \Longrightarrow \theta_s(t) \to \theta_p(t)$:

$$F_s(t) = -K \begin{pmatrix} \dot{\theta}_s(t) \\ \dot{\theta}_p(t) \\ \theta_s(t) - \theta_p(t) \end{pmatrix}. \tag{28}$$

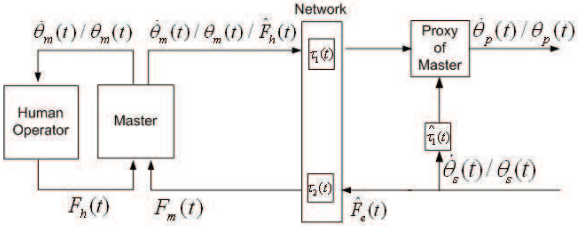
Control Scheme 3 - Master-Proxy Synchronization :



$$\begin{cases} \dot{x}_{mp}(t) & = A^0_{mp}x_{mp}(t) + A^1_{mp}x_{mp}(t-\tau_1(t)) + B_{mp}w_{mp}(t), \\ z_{mp}(t) & = C_{mp}x_{mp}(t). \end{cases} \quad (29)$$

$$x_{mp}(t) = \begin{pmatrix} \dot{\theta}_p(t) \\ \dot{\theta}_m(t) \\ \theta_p(t)-\theta_m(t) \end{pmatrix}, \quad w_{mp}(t) = \begin{pmatrix} \hat{F}_e(t-\hat{\tau}_1(t))+\hat{F}_h(t-\tau_1(t)) \\ F_m(t)+F_h(t) \end{pmatrix}, \qquad (30)$$
$$z_{mp}(t) = \begin{pmatrix} \theta_p(t)-\theta_m(t) \end{pmatrix}.$$

Control Scheme 3 - Master-Proxy Synchronization :

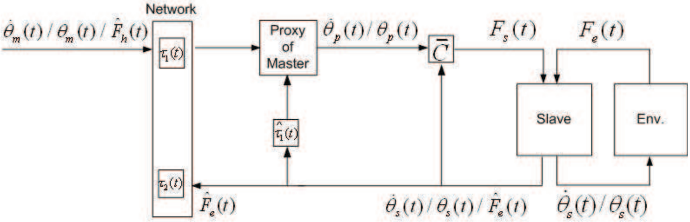


$$A^0_{mp} = \begin{pmatrix} A_m - B_m K^m_0 & 0 & 0 \\ 0 & A_m - B_m K^m_0 & 0 \\ 1 & -1 & 0 \end{pmatrix}, \quad B_{mp} = \begin{pmatrix} B_m & 0 \\ 0 & B_m \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} B^1_{mp} & B^2_{mp} \end{pmatrix},$$

$$C_{mp} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix},$$

$$A^1_{mp} = \begin{pmatrix} -B_m L_1 & -B_m L_2 & -B_m L_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -B^1_{mp} L. \tag{31}$$

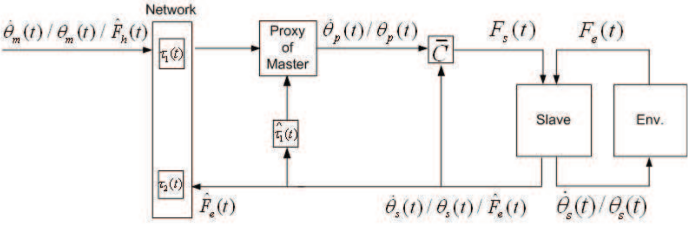
Control Scheme 3 - Proxy-Slave Synchronization :



$$\begin{cases} \dot{x}_{ps}(t) & = A_{ps}x_{ps}(t) + B_{ps}w_{ps}(t), \\ z_{ps}(t) & = C_{ps}x_{ps}(t), \end{cases} \tag{32}$$

$$x_{ps}(t) = \begin{pmatrix} \dot{\theta}_s(t) \\ \dot{\theta}_p(t) \\ \theta_s(t)-\theta_p(t) \end{pmatrix}, \quad z_{ps}(t) = \begin{pmatrix} \theta_s(t)-\theta_p(t) \end{pmatrix}, \tag{33}$$
$$w_{ps}(t) = \begin{pmatrix} F_e(t) \\ \hat{F}_e(t-\hat{\tau_1}(t)) + \hat{F}_h(t-\tau_1(t)) - F_p(t) \end{pmatrix}.$$

Control Scheme 3 - Proxy-Slave Synchronization :



$$
B_{ps} = \begin{pmatrix} B_s & 0 \\ 0 & B_m \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} B_{ps}^1 & B_{ps}^2 \end{pmatrix}, \quad C_{ps} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix},
$$

$$
\begin{aligned}
A_{ps} &= \begin{pmatrix} A_s - B_s K_0^s - B_s K_1 & -B_s K_2 & -B_s K_3 \\ 0 & A_m - B_m K_0^m & 0 \\ 1 & -1 & 0 \end{pmatrix} \\
&= \begin{pmatrix} A_s - B_s K_0^s & 0 & 0 \\ 0 & A_m - B_m K_0^m & 0 \\ 1 & -1 & 0 \end{pmatrix} + \begin{pmatrix} -B_s K_1 & -B_s K_2 & -B_s K_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
&= (A_{ps}^0 - B_{ps}^1 K).
\end{aligned} \tag{34}
$$

Control Scheme 3 - Global Performance Analysis :

$$\begin{cases} \dot{x}_{mps}(t) &= A^0_{mps}x_{mps}(t) + A^1_{mps}x_{mps}(t-\tau_1(t)) + B_{mps}w_{mps}(t), \\ z_{mps}(t) &= C_{mps}x_{mps}(t), \end{cases} \tag{35}$$

$$x_{mps}(t) = \begin{pmatrix} \dot{\theta}_s(t) \\ \dot{\theta}_p(t) \\ \dot{\theta}_m(t) \\ \theta_s(t)-\theta_p(t) \\ \theta_p(t)-\theta_m(t) \end{pmatrix}, \quad w_{mps}(t) = \begin{pmatrix} F_e(t) \\ \hat{F}_e(t-\hat{\tau}_1(t))+\hat{F}_h(t-\tau_1(t)) \\ F_m(t)+F_h(t) \end{pmatrix}, \quad z_{mps}(t) = \begin{pmatrix} \theta_s(t)-\theta_p(t) \\ \theta_p(t)-\theta_m(t) \end{pmatrix}. \tag{36}$$

$$A^0_{mps} = \begin{pmatrix} A_s-B_sK^s_0-B_sK_1 & -B_sK_2 & 0 & -B_sK_3 & 0 \\ 0 & A_m-B_mK^m_0 & 0 & 0 & 0 \\ 0 & 0 & A_m-B_mK^m_0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{pmatrix},$$

$$A^1_{mps} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -B_mL_1 & -B_mL_2 & 0 & -B_mL_3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad B_{mps} = \begin{pmatrix} B_s & 0 & 0 \\ 0 & B_m & 0 \\ 0 & 0 & B_m \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad C_{mps} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{37}$$

Conclusions :

☺ Control Scheme 1 - Bilateral state feedback control scheme ;
☺ Control Scheme 2 - Force-reflecting control scheme ;
☺ Control Scheme 3 - Force-reflecting proxy control scheme ;

Control Schema	Time Delays		Position Tracking	Force Tracking
	Constant	Time-varying		
1	√	√	√	
2	√	√	√	√
3	√	√	√	√

☺ Force estimation/measure : 2 & 3 ;
☺ Better performance, but additional computation load : 3 ;

2. Novel Control Schemes

☺ System Description & Assumptions ;
☺ Robust Stability Conditions ;
☺ Control Scheme 1 - Bilateral State Feedback Control Scheme [*B. Zhang et al., CCDC, 2011*] ;

[B. Zhang et al., CCDC, 2011] ;

☺ Control Scheme 2 - Force-Reflecting Control Scheme ;
☺ Control Scheme 3 - Force-Reflecting Proxy Control Scheme

[B. Zhang et al., ETFA, 2011] ;

☺ Results and Analysis ;

Simulation Conditions :

☺ Maximum amplitude of time-varying delays : $0.2s$;

☺ Master, proxy and slave models : $1/s$, $1/s$ and $2/s$; Poles : $[-100.0]$. $K_m^0 = 100$, $K_s^0 = 50$;

☺

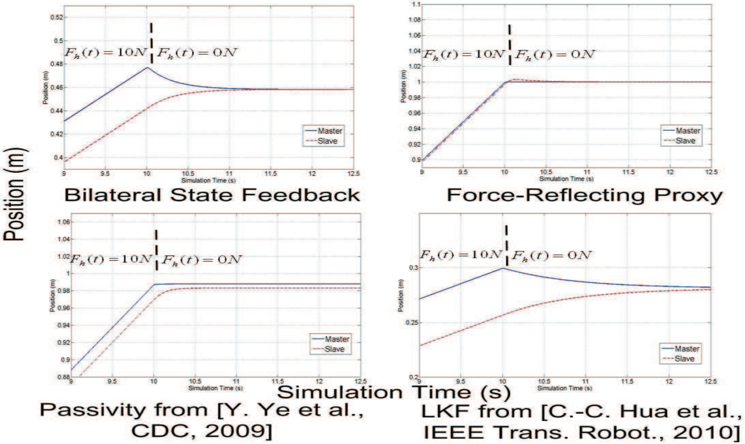
$$\begin{aligned} K_1 &= \begin{pmatrix} -0.1870 & -0.0368 & 65.0846 \end{pmatrix}, \\ K_2 &= \begin{pmatrix} 0.4419 & 0.0813 & -153.8704 \end{pmatrix}, \quad \gamma_{min}^{C_1/C_2} = 0.0123. \end{aligned} \tag{38}$$

☺

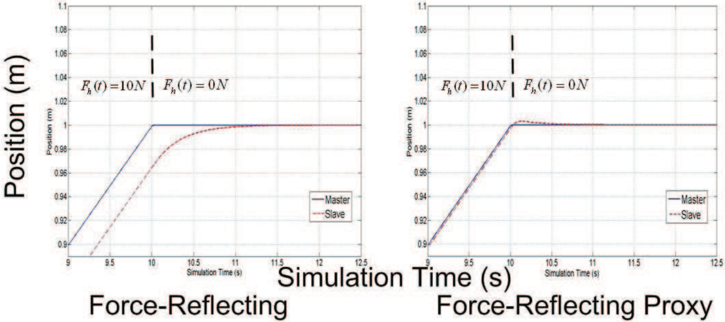
$$\begin{aligned} L &= \begin{pmatrix} -1.4566 & 0.1420 & 282.482 \end{pmatrix}, \quad \gamma_{min}^{L} = 0.0081, \\ K &= \begin{pmatrix} -29.9635 & -3.6393 & 618.536 \end{pmatrix}, \quad \gamma_{min}^{K} = 0.0075. \end{aligned} \tag{39}$$

Global stability of the system is verified with $\gamma_{min}^{g} = 0.0062$;

Abrupt Tracking Motion :



Abrupt Tracking Motion :

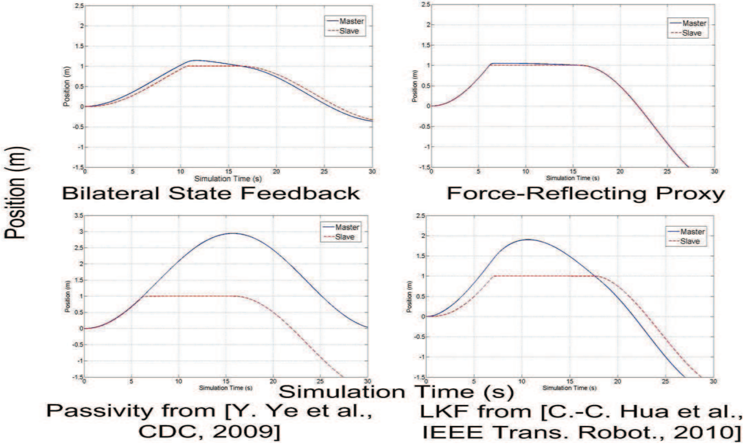


Wall Contact Motion :

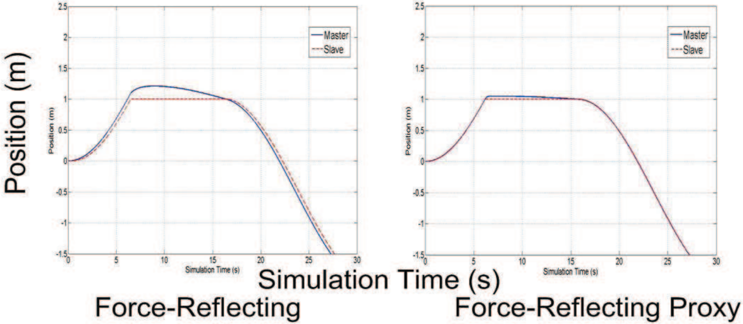
☺ Hard wall : the stiffness $K_e = 30kN/m$, the position $x = 1.0m$;
☺ Our aims :

1. When the slave robot reaches the wall, the master robot can stop as quickly as possible ;
2. When the slave robot returns after hitting the wall $(F_e(t) = 0)$, the system must restore the position tracking between the master and the slave ;
3. When the slave contacts the wall, the force tracking from the slave to the master can be assured ;

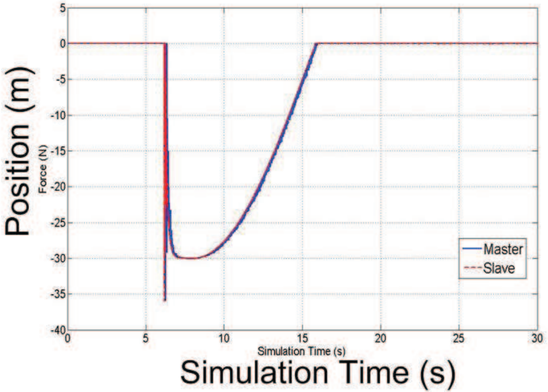
Wall Contact Motion :



Wall Contact Motion :

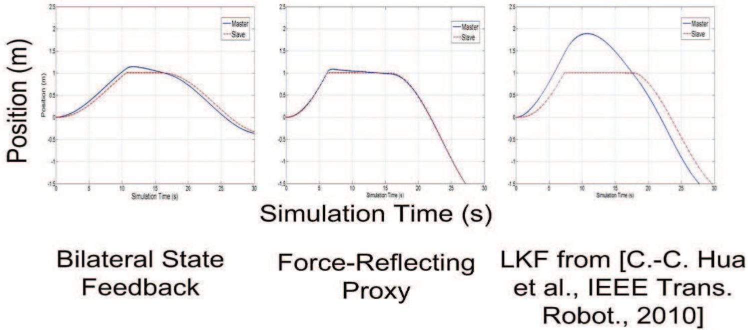


Wall Contact Motion :



☺ Force response in wall contact motion ($F_m(t)$; $\hat{F}_e(t)$).

Wall Contact Motion under Large Delays $h_2 = 1.0s$:



3. Robustness Aspects

☺ Discrete-Time System *[B. Zhang et al., TDS, 2012]* :

- *Discrete-Time Approach*;
- *Slave Controller Design*;
- *Results and Analysis*;

☺ Linear Parameter-Varying System (LPV) :

- *Polytopic-type uncertainties [B. Zhang et al., TDS, 2012]*;
- *Norm-bounded uncertainties [B. Zhang et al., ROCOND, 2012]*;
- *Results and Analysis*;

Discrete-Time System :

$$\begin{cases} x(k+1) = \sum_{i=0}^{q} A_i x(k-\tau_i(k)) + Bw(k), \\ z(k) = Cx(k). \end{cases} \tag{40}$$

Delay-Free Case :

$$\begin{cases} x(k+1) = A_0 x(k) + Bw(k), \\ z(k) = Cx(k). \end{cases} \tag{41}$$

Robust Stability Condition :

☺ Discrete LKF, $y(k) = x(k+1) - x(k)$:

$$V(x(k)) = x(k)^T P x(k) + \sum_{i=k-h_2}^{k-1} x(i)^T S_a x(i) + \sum_{i=k-h_1}^{k-1} x(i)^T S x(i) + h_1 \sum_{i=-h_1}^{-1} \sum_{j=k+i}^{k-1} y(j)^T R y(j) + \sum_{i=1}^{q} (h_2 - h_1) \sum_{j=-h_2}^{-h_1-1} \sum_{l=k+j}^{k-1} y(l)^T R_{ai} y(l). \tag{42}$$

☺ H_∞ control : $J(w) = \sum_{i=0}^{\infty} [z(k)^T z(k) - \gamma^2 w(k)^T w(k)] < 0$

☺ $\triangle V(x(k)) = V(x(k+1)) - V(x(k))$:

$$\triangle V(x(k)) + z(k)^T z(k) - \gamma^2 w(k)^T w(k) < 0. \tag{43}$$

☺ LMI condition is feasible (see manuscript) ;

☺ Rate-independent asymptotically stable and H_∞ performance $J(w) < 0$ for time-varying delays $\tau_i(k) \in [h_1, h_2]$, $h_2 \geq h_1 \geq 0$, $i = 1, 2, \ldots, q$;

3. Robustness Aspects

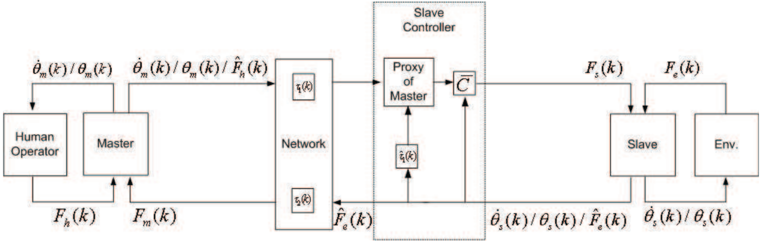
☺ Discrete-Time System *[B. Zhang et al., TDS, 2012]* :

- *Discrete-Time Approach* ;
- *Slave Controller Design* ;
- *Results and Analysis* ;

☺ Linear Parameter-Varying System (LPV) :

- *Polytopic-type uncertainties [B. Zhang et al., TDS, 2012]* ;
- *Norm-bounded uncertainties [B. Zhang et al., ROCOND, 2012]* ;
- *Results and Analysis* ;

System Description :



$$
\begin{aligned}
(\Sigma_m^d) \quad & x_m(k+1) = (A_{md} - B_{md}K_{md}^0)x_m(k) + B_{md}(F_m(k) + F_h(k)), \\
(\Sigma_s^d) \quad & x_s(k+1) = (A_{sd} - B_{sd}K_{sd}^0)x_s(k) + B_{sd}(F_s(k) + F_e(k)), \\
(\Sigma_p^d) \quad & x_p(k+1) = (A_{md} - B_{md}K_{md}^0)x_p(k) - B_{md}F_p(k) \\
& \quad + B_{md}(\hat{F}_e(k - \hat{\tau}_1(k)) + \hat{F}_h(k - \tau_1(k))).
\end{aligned} \tag{44}
$$

Slave Controller Design - Master-Proxy Synchronization :

☺ $L_d = \begin{pmatrix} L_{d1} & L_{d2} & L_{d3} \end{pmatrix}$:

$$F_p(k) = L_d \begin{pmatrix} \dot{\theta}_p(k-\hat{\tau}_1(k)) \\ \dot{\theta}_m(k-\tau_1(k)) \\ \theta_p(k-\hat{\tau}_1(k))-\theta_m(k-\tau_1(k)) \end{pmatrix}. \quad (45)$$

☺

$$(\Sigma_{mp}^d) \begin{cases} x_{mp}(k+1) & = A_{mpd}^0 x_{mp}(k) + A_{mpd}^1 x_{mp}(k-\tau_1(k)) \\ & \quad +B_{mpd} w_{mp}(k), \\ z_{mp}(k) & = C_{mpd} x_{mp}(k), \end{cases} \quad (46)$$

$A_{mpd}^1 \Longrightarrow L_d$;

Slave Controller Design - Proxy-Slave Synchronization :

☺ $K_d = \begin{pmatrix} K_{d1} & K_{d2} & K_{d3} \end{pmatrix}$:

$$F_s(k) = -K_d \begin{pmatrix} \dot{\theta}_s(k) \\ \dot{\theta}_p(k) \\ \theta_s(k) - \theta_p(k) \end{pmatrix}. \tag{47}$$

☺

$$(\Sigma_{ps}^d) \begin{cases} x_{ps}(k+1) & = A_{psd} x_{ps}(k) + B_K F_s(k) + B_{psd} w_{ps}(k), \\ z_{ps}(k) & = C_{psd} x_{ps}(k), \end{cases} \tag{48}$$

$$\Downarrow$$

$$(\bar{\Sigma}_{ps}^d) \begin{cases} x_{ps}(k+1) & = (A_{psd} - B_K K_d) x_{ps}(k) + B_{psd} w_{ps}(k), \\ z_{ps}(k) & = C_{psd} x_{ps}(k). \end{cases} \tag{49}$$

Slave Controller Design - Global Performance Analysis :

$$(\Sigma^d_{mps}) \begin{cases} x(k+1) & = A_{mpsd}x(k) + B^K_{mpsd}F_s(k) - B^L_{mpsd}F_p(k) + B_{mpsd}w(k), \\ z(k) & = C^{mpsd}x(k), \end{cases} \quad (50)$$

$$\begin{aligned} F_s(k) &= -\bar{K}_d x(k) = -\begin{pmatrix} K_{d1} & K_{d2} & 0 & K_{d3} & 0 \end{pmatrix} x(k), \\ F_p(k) &= \bar{L}_d x(k-\tau_1(k)) = \begin{pmatrix} 0 & L_{d1} & L_{d2} & 0 & L_{d3} \end{pmatrix} x(k-\tau_1(k)), \\ B^K_{mpsd} &= \begin{pmatrix} B_{sd} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad B^L_{mpsd} = \begin{pmatrix} 0 \\ B_{md} \\ 0 \\ 0 \\ 0 \end{pmatrix}, \end{aligned} \quad (51)$$

$$\Downarrow$$

$$(\bar{\Sigma}^d_{mps}) \begin{cases} x(k+1) & = A^d_0 x(k) + A^d_1 x(k-\tau_1(k)) + B_{mpsd}w(k), \\ z(k) & = C^{mpsd}x(k), \end{cases} \quad (52)$$

$A^d_0 = A_{mpsd} - B^K_{mpsd}\bar{K}_d$, $A^d_1 = -B^L_{mpsd}\bar{L}_d$.

3. Robustness Aspects

☺ Discrete-Time System *[B. Zhang et al., TDS, 2012]* :

- *Discrete-Time Approach* ;
- *Slave Controller Design* ;
- *Results and Analysis* ;

☺ Linear Parameter-Varying System (LPV) :

- *Polytopic-type uncertainties [B. Zhang et al., TDS, 2012]* ;
- *Norm-bounded uncertainties [B. Zhang et al., ROCOND, 2012]* ;
- *Results and Analysis* ;

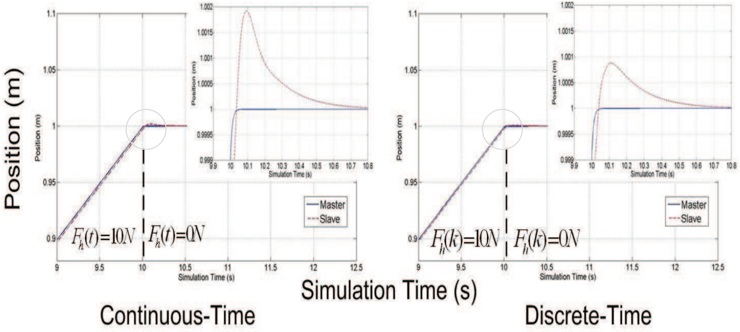
Simulation Conditions :

☺ $T = 0.001s$, $h_1 = 1$, $h_2 = 100$ (in continuous-time domain, $h_1 = 0.001s$, $h_2 = 0.1s$);

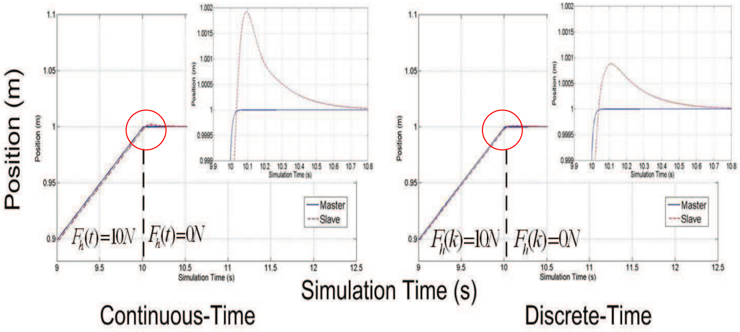
☺

$$\begin{aligned} &L_d = \begin{pmatrix} 4.6815 & -5.1390 & 540.7828 \end{pmatrix}, \quad \gamma_{min}^{L_d} = 0.0051, \\ &K_d = \begin{pmatrix} 273 & -127 & 10961 \end{pmatrix}, \quad \gamma_{min}^{K_d} = 2.9568 \times 10^{-4}, \\ &\gamma_{min}^{g} = 0.0327. \end{aligned} \tag{53}$$

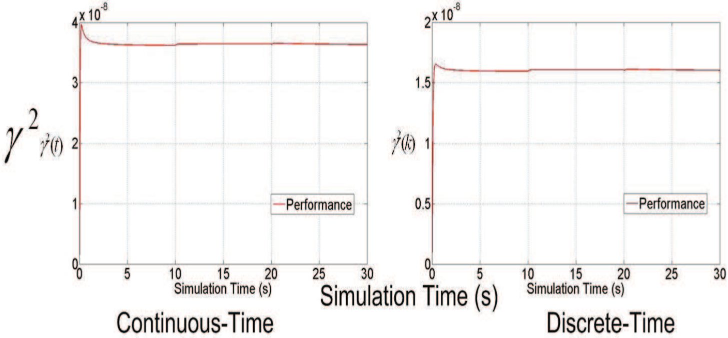
Abrupt Tracking Motion :



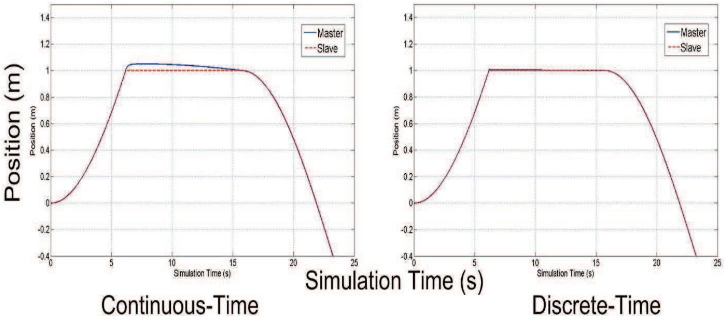
Abrupt Tracking Motion :



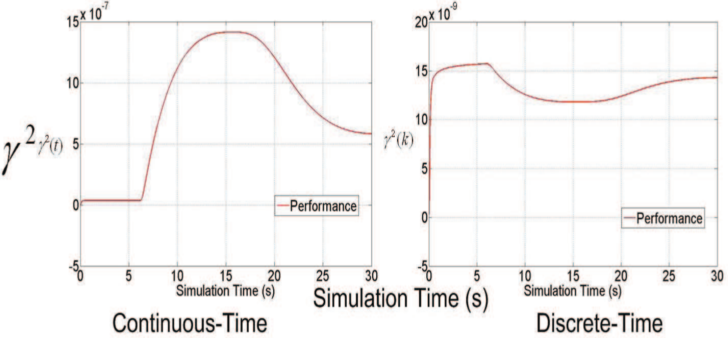
Abrupt Tracking Motion :



Wall Contact Motion :



Wall Contact Motion :



3. Robustness Aspects

☺ Discrete-Time System *[B. Zhang et al., TDS, 2012]* :

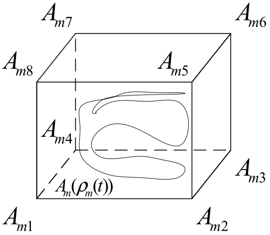
- *Discrete-Time Approach* ;
- *Slave Controller Design* ;
- *Results and Analysis* ;

☺ Linear Parameter-Varying System (LPV) :

- *Polytopic-type uncertainties [B. Zhang et al., TDS, 2012]* ;
- *Norm-bounded uncertainties [B. Zhang et al., ROCOND, 2012]* ;
- *Results and Analysis* ;

System Description :

$$
\begin{aligned}
&(\Sigma_m)\dot{x}_m(t) = (A_m(\rho_m(t)) - B_m(\rho_m(t))K_m^0)x_m(t) + B_m(\rho_m(t))(F_m(t) + F_h(t)),\\
&(\Sigma_s)\dot{x}_s(t) = (A_s(\rho_s(t)) - B_s(\rho_s(t))K_s^0)x_s(t) + B_s(\rho_s(t))(F_s(t) + F_e(t)),\\
&(\Sigma_p)\dot{x}_p(t) = (A_m(\rho_p(t)) - B_m(\rho_p(t))K_m^0)x_p(t) + B_m(\rho_p(t))(\hat{F}_e(t - \hat{\tau}_1(t)) + \hat{F}_h(t - \tau_1(t)) - F_p(t)),\\
&[A_m(\rho_m(t)), B_m(\rho_m(t))] = \sum_{j=1}^{N} \rho_{mj}(t)[A_{mj}, B_{mj}], \quad [A_s(\rho_s(t)), B_s(\rho_s(t))] = \sum_{j=1}^{N} \rho_{sj}(t)[A_{sj}, B_{sj}],\\
&[A_m(\rho_p(t)), B_m(\rho_p(t))] = \sum_{j=1}^{N} \rho_{pj}(t)[A_{mj}, B_{mj}].
\end{aligned}
\tag{54}
$$



Controller Design - Problem 1 :

K_m^0 & K_s^0 : robust stability *w.r.t* polytopic-type uncertainties ;

Controller Design - Problem 2 :

L & K : stability & position/force tracking *w.r.t* time-varying delays & polytopic-type uncertainties ;

Robust Stability Theorem *[E. Fridman, IMA Journal of Mathematical Control and Information, 2006]* :

☺ $P > 0$, $R > 0$, $S > 0$, $S_a > 0$, $R_{ai} > 0$, P_2, P_3, Y_1, Y_2, $i = 1, 2, ..., q$, and a positive scalar $\gamma > 0$;
☺ N LMI conditions are feasible (see manuscript) ;
☺ Rate-independent asymptotically stable with H_∞ performance $J(w) < 0$ for time-varying delays $\tau_i(t) \in [h_1, h_2]$, $i = 1, 2, ..., q$;

Problem 2 Slave Controller Design

☺ Master-proxy synchronization :

$$\begin{cases} \dot{x}_{mp}(t) & = A^0_{mp}(\rho_{mp}(t))x_{mp}(t) + A^1_{mp}(\rho_{mp}(t))x_{mp}(t-\tau_1(t)) + B_{mp}(\rho_{mp}(t))w_{mp}(t), \\ z_{mp}(t) & = C_{mp}x_{mp}(t), \end{cases} \tag{55}$$

K^0_m & $K^0_s \Longrightarrow A^0_{mp}(\rho_{mp}(t))$; $A^1_{mp}(\rho_{mp}(t)) \Longrightarrow L$;

☺ Proxy-slave synchronization :

$$\begin{cases} \dot{x}_{ps}(t) & = A_{ps}(\rho_{ps}(t))x_{ps}(t) + B_{ps}(\rho_{ps}(t))w_{ps}(t), \\ z_{ps}(t) & = C_{ps}x_{ps}(t), \end{cases} \tag{56}$$

$A_{ps}(\rho_{ps}(t)) \Longrightarrow K$;

☺ Global performance analysis with K^0_m & K^0_s, L & K :

$$\begin{cases} \dot{x}_{mps}(t) & = A^0_{mps}(\rho_{mps}(t))x_{mps}(t) + A^1_{mps}(\rho_{mps}(t))x_{mps}(t-\tau_1(t)) \\ & \quad + B_{mps}(\rho_{mps}(t))w_{mps}(t), \\ z_{mps}(t) & = C_{mps}x_{mps}(t). \end{cases} \tag{57}$$

Remark - Nonlinear Systems :

$$
\begin{aligned}
(\Sigma_m)\quad & M_m(\theta_m)\ddot{\theta}_m(t) + C_m(\theta_m,\dot{\theta}_m)\dot{\theta}_m(t) + g_m(\theta_m) = F_h(t) + F_m(t),\\
(\Sigma_p)\quad & M_m(\theta_p)\ddot{\theta}_p(t) + C_m(\theta_p,\dot{\theta}_p)\dot{\theta}_p(t) + g_m(\theta_p) = \hat{F}_e(t-\hat{\tau}_1(t)) + \hat{F}_h(t-\tau_1(t)) - F_p(t),\\
(\Sigma_s)\quad & M_s(\theta_s)\ddot{\theta}_s(t) + C_s(\theta_s,\dot{\theta}_s)\dot{\theta}_s(t) + g_s(\theta_s) = F_e(t) + F_s(t),
\end{aligned}
\tag{58}
$$

$$\Downarrow$$

$$
\begin{aligned}
(\bar{\Sigma}_m)\quad & \ddot{\theta}_m(t) = A_m(\theta_m,\dot{\theta}_m)\dot{\theta}_m(t) + B_m(\theta_m)(F_h(t) + F_m(t) - g_m(\theta_m)),\\
(\bar{\Sigma}_p)\quad & \ddot{\theta}_p(t) = A_m(\theta_p,\dot{\theta}_p)\dot{\theta}_p(t) + B_m(\theta_p)(\hat{F}_e(t-\hat{\tau}_1(t)) + \hat{F}_h(t-\tau_1(t)) - F_p(t) - g_m(\theta_p)),\\
(\bar{\Sigma}_s)\quad & \ddot{\theta}_s(t) = A_s(\theta_s,\dot{\theta}_s)\dot{\theta}_s(t) + B_s(\theta_s)(F_e(t) + F_s(t) - g_s(\theta_s)),
\end{aligned}
\tag{59}
$$

$$
\begin{aligned}
A_m(\theta_m,\dot{\theta}_m) &= -M_m^{-1}(\theta_m)C_m(\theta_m,\dot{\theta}_m), \quad B_m(\theta_m) = M_m^{-1}(\theta_m),\\
A_m(\theta_p,\dot{\theta}_p) &= -M_m^{-1}(\theta_p)C_m(\theta_p,\dot{\theta}_p), \quad B_m(\theta_p) = M_m^{-1}(\theta_p),\\
A_s(\theta_s,\dot{\theta}_s) &= -M_s^{-1}(\theta_s)C_s(\theta_s,\dot{\theta}_s), \quad B_s(\theta_s) = M_s^{-1}(\theta_s).
\end{aligned}
\tag{60}
$$

3. Robustness Aspects

☺ Discrete-Time System *[B. Zhang et al., TDS, 2012]* :

- *Discrete-Time Approach* ;
- *Slave Controller Design* ;
- *Results and Analysis* ;

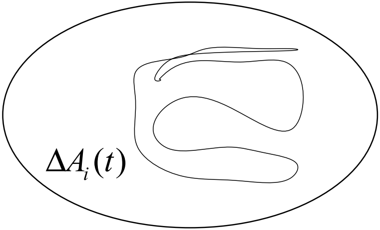
☺ Linear Parameter-Varying System (LPV) :

- *Polytopic-type uncertainties* *[B. Zhang et al., TDS, 2012]* ;
- *Norm-bounded uncertainties* *[B. Zhang et al., ROCOND, 2012]* ;
- *Results and Analysis* ;

System Description :

$$
\begin{aligned}
(\Sigma_m) \quad \dot{x}_m(t) &= ((A_m + \Delta A_m(t)) - (B_m + \Delta B_m(t))K_m^0)x_m(t) \\
&\quad + (B_m + \Delta B_m(t))(F_m(t) + F_h(t)), \\
(\Sigma_s) \quad \dot{x}_s(t) &= ((A_s + \Delta A_s(t)) - (B_s + \Delta B_s(t))K_s^0)x_s(t) \\
&\quad + (B_s + \Delta B_s(t))(F_s(t) + F_e(t)), \\
(\Sigma_p) \quad \dot{x}_p(t) &= ((A_m + \Delta A_p(t)) - (B_m + \Delta B_p(t))K_m^0)x_p(t) \\
&\quad - (B_m + \Delta B_p(t))F_p(t) \\
&\quad + (B_m + \Delta B_p(t))(\hat{F}_e(t - \hat{\tau}_1(t)) + \hat{F}_h(t - \tau_1(t))),
\end{aligned}
\tag{61}
$$

$$
i = \{m, s, p\} : \quad \Delta A_i(t) = G_i \Delta(t) D_i, \quad \Delta B_i(t) = H_i \Delta(t) E_i, \quad \Delta(t)^T \Delta(t) \leqslant I. \tag{62}
$$



System Description :

$$\begin{aligned}
(\Sigma_m) \quad \dot{x}_m(t) &= ((A_m + \Delta A_m(t)) - (B_m + \Delta B_m(t))K_m^0)x_m(t) \\
&+ (B_m + \Delta B_m(t))(F_m(t) + F_h(t)), \\
(\Sigma_s) \quad \dot{x}_s(t) &= ((A_s + \Delta A_s(t)) - (B_s + \Delta B_s(t))K_s^0)x_s(t) \\
&+ (B_s + \Delta B_s(t))(F_s(t) + F_e(t)), \\
(\Sigma_p) \quad \dot{x}_p(t) &= ((A_m + \Delta A_p(t)) - (B_m + \Delta B_p(t))K_m^0)x_p(t) \\
&- (B_m + \Delta B_p(t))F_p(t) \\
&+ (B_m + \Delta B_p(t))(\hat{F}_e(t - \hat{\tau}_1(t)) + \hat{F}_h(t - \tau_1(t))),
\end{aligned} \tag{63}$$

$$i = \{m, s, p\} : \quad \Delta A_i(t) = G_i \Delta(t) D_i, \quad \Delta B_i(t) = H_i \Delta(t) E_i, \quad \Delta(t)^T \Delta(t) \leqslant I. \tag{64}$$

Slave Controller Design Solution :

Transformation from norm-bounded uncertain system to linear time-delay system ;

Problem 2 Slave Controller Design - Master-Proxy Synchronization :

$$\begin{cases} \dot{x}_{mp}(t) & = (A^0_{mp} + \Delta A^0_{mp}(t))x_{mp}(t) + (A^1_{mp} + \Delta A^1_{mp}(t))x_{mp}(t-\tau_1(t)) \\ & +(B_{mp} + \Delta B_{mp}(t))w_{mp}(t), \\ z_{mp}(t) & = C_{mp}x_{mp}(t), \end{cases} \tag{65}$$

K^0_m & $K^0_s \Longrightarrow A^0_{mp} + \Delta A^0_{mp}(t)$;

$$\Downarrow$$

$$\begin{aligned} \varphi_p(t) &= (\Delta A_p(t) - \Delta B_p(t)K^0_m)\dot{\theta}_p(t) + \Delta B_p(t)(\hat{F}_e(t-\hat{\tau}_1(t)) + \hat{F}_h(t-\tau_1(t))), \\ \varphi_m(t) &= (\Delta A_m(t) - \Delta B_m(t)K^0_m)\dot{\theta}_m(t) + \Delta B_m(t)(F_m(t) + F_h(t)), \\ \mu_p(t) &= -\Delta B_p(t)Lx_{mp}(t-\tau_1(t)). \end{aligned} \tag{66}$$

$$\begin{cases} \dot{x}_{mp}(t) & = A^0_{mp}x_{mp}(t) + A^1_{mp}x_{mp}(t-\tau_1(t)) + \widetilde{B}_{mp}\widetilde{w}_{mp}(t), \\ z_{mp}(t) & = C_{mp}x_{mp}(t), \end{cases} \tag{67}$$

$$\widetilde{w}(t) = \begin{pmatrix} B_m\hat{F}_e(t-\hat{\tau}_1(t)) + B_m\hat{F}_h(t-\tau_1(t)) + \varphi_p(t) + \mu_p(t) \\ B_mF_m(t) + B_mF_h(t) + \varphi_m(t) \end{pmatrix}, \quad \widetilde{B}_{mp} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \tag{68}$$

$A^1_{mp} \Longrightarrow L$;

Slave Controller Design - Proxy-Slave Synchronization :

$$\begin{cases} \dot{x}_{ps}(t) &= (A_{ps} + \Delta A_{ps}(t))x_{ps}(t) + (B_{ps} + \Delta B_{ps}(t))w_{ps}(t), \\ z_{ps}(t) &= C_{ps}x_{ps}(t), \end{cases} \tag{69}$$

$A_{ps} + \Delta A_{ps}(t) \Longrightarrow K$;

$$\Downarrow$$

$$\varphi_s(t) = (\Delta A_s(t) - \Delta B_s(t)K_s^0)\dot{\theta}_s(t) + \Delta B_s(t)F_e(t),$$
$$\mu_s(t) = -\Delta B_{ps}^1(t)Kx_{ps}(t). \tag{70}$$

$$\begin{cases} \dot{x}_{ps}(t) &= A_{ps}x_{ps}(t) + \widetilde{B}_{ps}\widetilde{w}_{ps}(t), \\ z_{ps}(t) &= C_{ps}x_{ps}(t), \end{cases} \tag{71}$$

$$\widetilde{w}_{ps}(t) = \begin{pmatrix} B_sF_e(t)+\varphi_s(t)+\mu_s(t) \\ B_m\hat{F}_e(t-\hat{\tau}_1(t))+B_m\hat{F}_h(t-\tau_1(t))-B_mF_p(t)+\varphi_p(t)+\mu_p(t) \end{pmatrix}, \quad \widetilde{B}_{ps} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \tag{72}$$

$A_{ps} \Longrightarrow K$;

Slave Controller Design - Global Performance Analysis :

$$\begin{cases} \dot{x}_{mps}(t) & = (A^0_{mps} + \Delta A^0_{mps}(t))x_{mps}(t) + (A^1_{mps} + \Delta A^1_{mps}(t))x_{mps}(t-\tau_1(t)) \\ & \quad +(B_{mps} + \Delta B_{mps}(t))w_{mps}(t), \\ z_{mps}(t) & = C_{mps}x_{mps}(t). \end{cases} \tag{73}$$

$$\Downarrow$$

$$\begin{cases} \dot{x}_{mps}(t) & = A^0_{mps}x_{mps}(t) + A^1_{mps}x_{mps}(t-\tau_1(t)) + \widetilde{B}_{mps}\widetilde{w}_{mps}(t), \\ z_{mps}(t) & = C_{mps}x_{mps}(t), \end{cases} \tag{74}$$

$$\widetilde{w}_{mps}(t) = \begin{pmatrix} B_s F_e(t) + \varphi_s(t) \\ B_m \hat{F}_e(t-\hat{\tau}_1(t)) + B_m \hat{F}_h(t-\tau_1(t)) + \varphi_p(t) \\ B_m F_m(t) + B_m F_h(t) + \varphi_m(t) \end{pmatrix}, \quad \widetilde{B}_{mps} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{75}$$

3. Robustness Aspects

☺ Discrete-Time System *[B. Zhang et al., TDS, 2012]* :

- *Discrete-Time Approach* ;
- *Slave Controller Design* ;
- *Results and Analysis* ;

☺ Linear Parameter-Varying System (LPV) :

- *Polytopic-type uncertainties* *[B. Zhang et al., TDS, 2012]* ;
- *Norm-bounded uncertainties* *[B. Zhang et al., ROCOND, 2012]* ;
- *Results and Analysis* ;

Simulation Conditions :

☺

$$\begin{aligned} polytopic-type: \quad & A_m(\rho_m(t)) = A_s(\rho_s(t)) = 0, \\ & B_m(\rho_m(t)) = B_s(\rho_s(t)) = \frac{1}{\rho(t)}, \quad \rho(t) \in [0.5, 1], \\ norm-bounded-type: \quad & A_m = A_s = 0, \quad G_i = D_i = 0, \\ & B_m = B_s = 1.5, \quad H_i = 0.5, \quad E_i = 1, \quad i = \{m, p, s\}. \end{aligned} \tag{76}$$

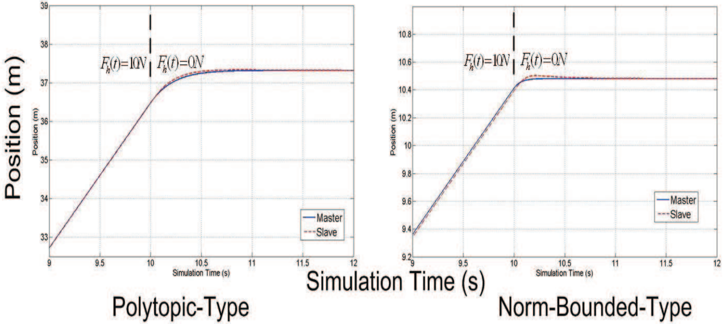
☺

$$\begin{aligned} polytopic-type: \quad & K_m^0 = 2.6585, \quad K_s^0 = 2.6585, \\ norm-bounded-type: \quad & K_m^0 = 9.5117, \quad K_s^0 = 3.3812. \end{aligned} \tag{77}$$

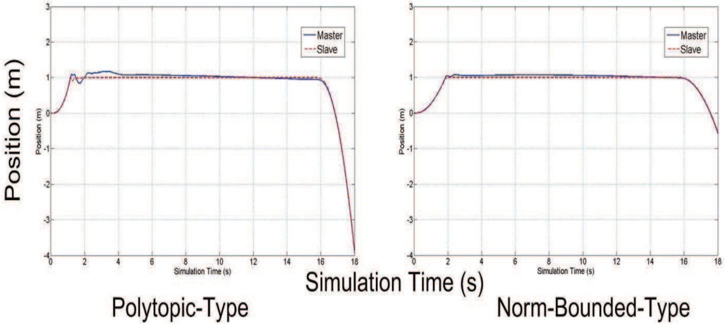
☺

$$\begin{aligned} polytopic-type: \quad & L = \begin{pmatrix} 1.3218 & -1.3219 & 6.3602 \end{pmatrix}, \quad \gamma_{min}^L = 0.4436, \\ & K = \begin{pmatrix} 20.4799 & -21.2537 & 575.2051 \end{pmatrix}, \quad \gamma_{min}^K = 0.0164, \\ & \gamma_{min}^g = 0.4595, \\ norm-bounded-type: \quad & L = \begin{pmatrix} 1.6218 & -1.6264 & 29.5012 \end{pmatrix}, \quad \gamma_{min}^L = 0.05, \\ & K = \begin{pmatrix} 16.4993 & -12.0059 & 434.4988 \end{pmatrix}, \quad \gamma_{min}^K = 0.0072, \\ & \gamma_{min}^g = 0.0376. \end{aligned} \tag{78}$$

Abrupt Tracking Motion :



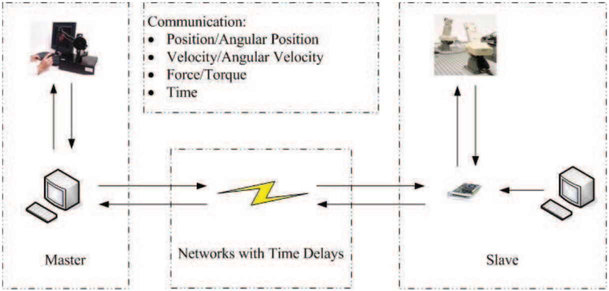
Wall Contact Motion :



4. Experimentations

- ☺ Experimental Test-Bench ;
- ☺ Force Estimation ;
- ☺ Results and Analysis ;

Experimental Test-Bench :

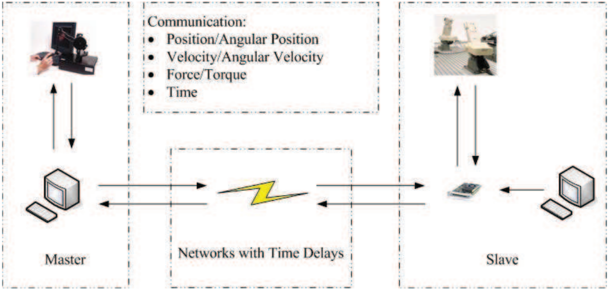


Master, Phantom Premium 1.0A :

$$\ddot{\theta}_m(t) = -\frac{\dot{\theta}_m(t)}{\tau_m} + \frac{K_m}{\tau_m} u_m(t), \tag{79}$$

$\tau_m = 0.448s$, $K_m = 0.0176s/kg$;

Experimental Test-Bench :

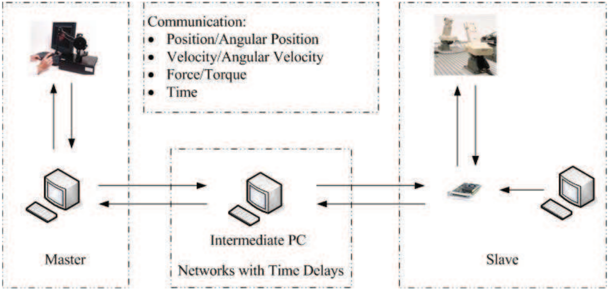


Slave, Mitsubishi Robot + CRIO card :

$$\ddot{\theta}_s(t) = -\frac{\dot{\theta}_s(t)}{\tau_s} + \frac{K_s}{\tau_s} u_s(t) - F_s sign(\dot{\theta}_s(t)), \qquad (80)$$

$\tau_s = 0.32s$, $K_s = 1.85s/kg$, $F_s = 0.30m/s^2$;

Experimental Test-Bench :



Networks : UDP ;

4. Experimentations

- ☺ Experimental Test-Bench ;
- ☺ Force Estimation ;
- ☺ Results and Analysis ;

Force Estimation $F_h(t)$:

☺

$$\dot{x}_m(t) = (A_m - B_m K_m^0) x_m(t) + B_m(F_m(t) + F_h(t)). \tag{81}$$

☺ $F_h^{(n+1)}(t) = 0$ for some n :

$$\dot{\varepsilon}_h(t) = A_h \varepsilon_h(t) + B_h F_m(t), \tag{82}$$

$$\varepsilon_h(t) = \begin{pmatrix} x_m(t) \\ F_h(t) \\ \dot{F}_h(t) \\ \vdots \\ F_h^{(n)}(t) \end{pmatrix}, \quad A_h = \begin{pmatrix} A_m - B_m K_m^0 & B_m & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & I \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}, \quad B_h = \begin{pmatrix} B_m \\ 0 \\ \vdots \\ 0 \end{pmatrix}. \tag{83}$$

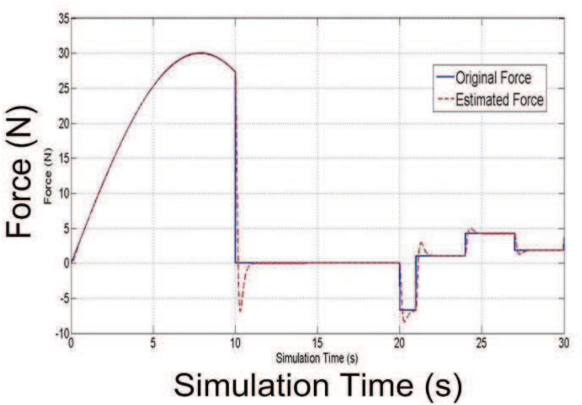
☺ $y_h(t) = x_m(t) = C_h \varepsilon_h(t)$:

$$\dot{\hat{\varepsilon}}_h(t) = A_h \hat{\varepsilon}_h(t) + B_h F_m(t) + L_h(y_h(t) - \hat{y}_h(t)). \tag{84}$$

☺ $e_h(t) = \varepsilon_h(t) - \hat{\varepsilon}_h(t)$:

$$\dot{e}_h(t) = (A_h - L_h C_h) e_h(t). \tag{85}$$

Force Estimation $F_h(t)$:

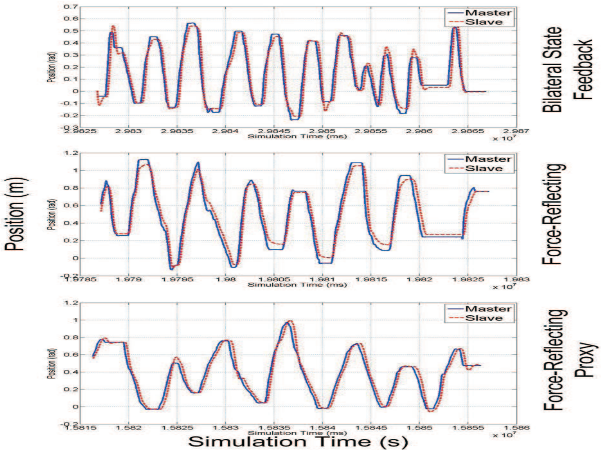


☺ $n = 1$, the eigenvalues of Luenberger observer as $[-11, -10, -9]$;

4. Experimentations

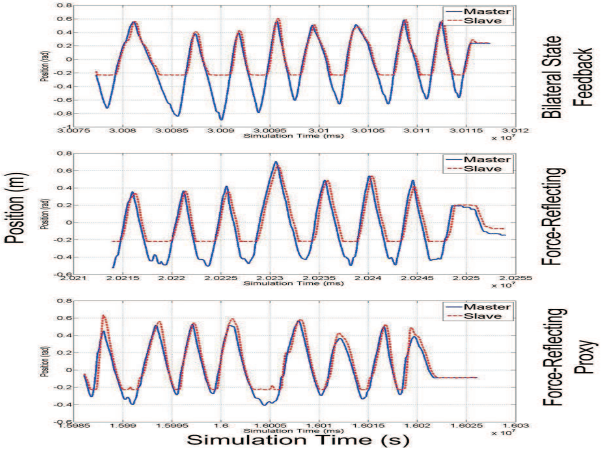
- ☺ Experimental Test-Bench ;
- ☺ Force Estimation ;
- ☺ Results and Analysis ;

Abrupt Tracking Motion :

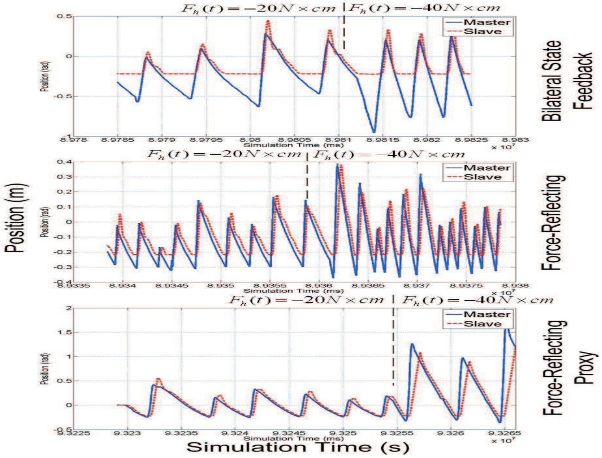


☺ $h_2 = 0.3s$;

Wall Contact Motion :



Wall Contact Motion under Constant $F_h(t)$:



5. Conclusions & Perspectives

Conclusions :

☺ 3 novel control schemes ($3 \geqslant 2 \geqslant 1$) :

1. Bilateral state feedback control scheme (Position/Velocity-Position/Velocity) ;
2. Force-reflecting control scheme (Position/Velocity-Force) ;
3. Force-reflecting proxy control scheme (Position/Velocity/Force-Force) ;

☺ Formal proofs :

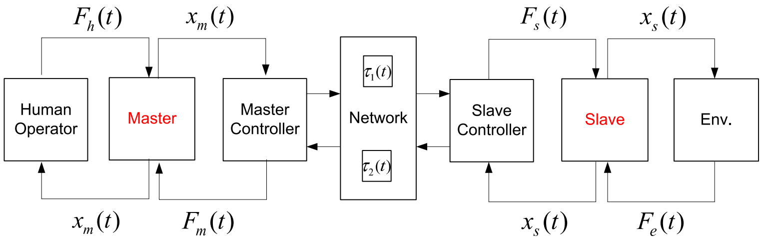
1. Stability (LKF) ;
2. Performance (H_∞ control) ;
3. Time-varying (large) delays ;
4. Uncertainties (polytopic or norm bounded) ;
5. Continuous or discrete time ;

☺ LMI design of the controllers ;

☺ Simulations and experimentations :

1. Abrupt tracking motion ;
2. Wall contact motion ;

Perspectives :

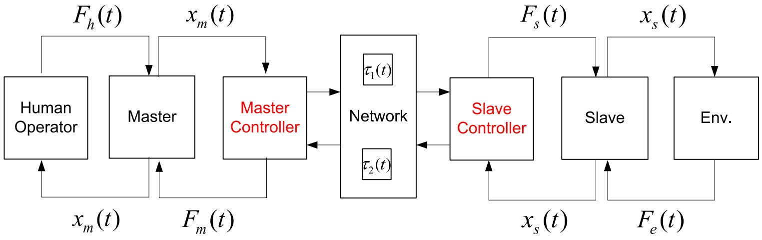


☺ Nonlinear systems *e.g.* master :

$$M(\theta)\ddot{\theta}(t) + C(\theta, \dot{\theta})\dot{\theta}(t) + g(\theta) = F_h(t) + F_m(t). \tag{86}$$

☺ One master/multiple slaves teleoperation system ;

Perspectives :

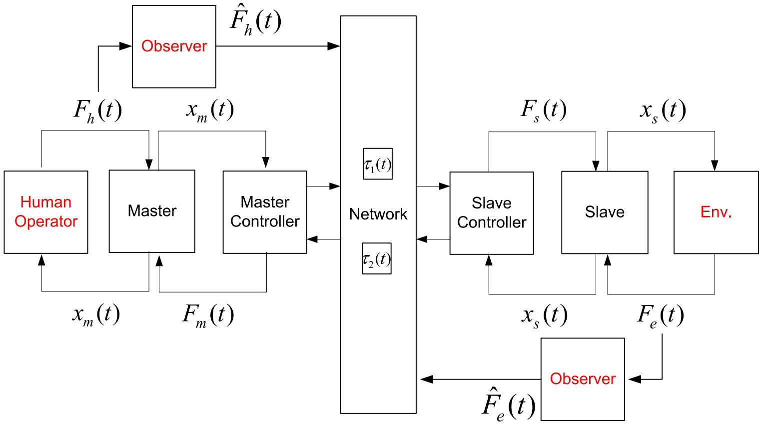


☺ Delay-scheduled state-feedback controller design :

$$K(\tau(t)) = K_0 + K_1\tau(t) + K_2\tau^2(t) + \cdots + K_h\tau^h(t) = \sum_{i=0}^{h} K_i\tau^i(t). \tag{87}$$

☺ Switch control strategy between the passivity-based approach and the approach proposed in this thesis ;

Perspectives :



- ☺ Global stability analysis with perturbation observers ;
- ☺ Dynamics of the human operator and the environment ;

Thank you for your attention ! ☺